

Got it. Below are **exam-ready 10-mark long-answer questions with derivations** for your **EMTL Unit-1**, using the content from your notes and syllabus.

UNIT-1 EMTL — 10 MARK QUESTIONS WITH ANSWERS

Q1. Explain electromagnetic field and write the basic introduction to electromagnetic fields.

Answer:

Electromagnetic field theory deals with the study of electric and magnetic fields produced by electric charges, either at rest or in motion.

1. Electric field

If charges are at rest, they produce an **electric field**.

The electric field intensity is defined as force per unit charge:

$$E = \frac{F}{Q}$$

The unit of electric field intensity is:

$$V/m \quad \text{or} \quad N/C$$

For a point charge Q , the electric field intensity is:

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R$$

where,

$$\epsilon_0 = \frac{10^{-9}}{36\pi} F/m$$

is the permittivity of free space.

2. Magnetic field

If charges are in motion, they produce a **magnetic field**.

The magnetic field intensity is represented by H .

Magnetic flux density B is related to magnetic field intensity by:

$$B = \mu H$$

where μ is the permeability of the medium.

For free space:

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

3. Time-varying fields

If electric and magnetic fields vary with time, then they are called **time-varying electromagnetic fields**.

A time-varying electric field produces magnetic field and a time-varying magnetic field produces electric field.

This concept is explained by Maxwell's equations.

4. Applications of electromagnetic fields

Electromagnetic fields are used in:

1. Antennas

2. Microwaves
 3. Satellite communication
 4. Remote sensing
 5. Transmission lines
 6. Electrical machines
 7. Communication systems
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5. Difference between circuit theory and electromagnetic theory

Circuit theory mainly deals with:

$$V, I, R, L, C$$

whereas electromagnetic theory mainly deals with:

$$E, D, H, B$$

Thus, EMTL gives field-level explanation of electrical and magnetic phenomena.

Q2. State and derive Maxwell's equations in electrostatic field.

Answer:

In electrostatics, charges are at rest and fields do not vary with time.

The two important Maxwell equations for electrostatic field are:

$$\nabla \cdot D = \rho_v$$

and

$$\nabla \times E = 0$$

1. Gauss law

Gauss law states that the total electric flux through a closed surface is equal to the total charge enclosed by that surface.

$$\psi = Q_{enc}$$

Electric flux is:

$$\psi = \oint_S D \cdot ds$$

Therefore,

$$\oint_S D \cdot ds = Q_{enc}$$

But enclosed charge is:

$$Q_{enc} = \int_V \rho_v dv$$

Hence,

$$\oint_S D \cdot ds = \int_V \rho_v dv$$

Using divergence theorem:

$$\oint_S D \cdot ds = \int_V \nabla \cdot D dv$$

Therefore,

$$\int_V \nabla \cdot D dv = \int_V \rho_v dv$$

Since the volume is arbitrary,

$$\boxed{\nabla \cdot D = \rho_v}$$

This is Maxwell's first equation in point form.

2. Curl of electrostatic field

In electrostatic field, potential difference is independent of path.

For a closed path:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

Using Stokes theorem:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s}$$

Therefore,

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = 0$$

Hence,

$$\boxed{\nabla \times \mathbf{E} = 0}$$

This shows that electrostatic field is conservative or irrotational.

Final electrostatic Maxwell equations

$$\boxed{\nabla \cdot \mathbf{D} = \rho_v}$$

$$\boxed{\nabla \times \mathbf{E} = 0}$$

Q3. State and derive Maxwell's equations for magnetostatic field.

Answer:

In magnetostatic field, charges are moving with constant velocity and current is steady.

The field does not vary with time.

The two important Maxwell equations are:

$$\nabla \times H = J$$

and

$$\nabla \cdot B = 0$$

1. Ampere's circuit law

Ampere's circuit law states that the line integral of magnetic field intensity around a closed path is equal to the current enclosed by the path.

$$\oint_C H \cdot dl = I_{enc}$$

Current enclosed can be written as surface integral of current density:

$$I_{enc} = \int_S J \cdot ds$$

Therefore,

$$\oint_C H \cdot dl = \int_S J \cdot ds$$

Using Stokes theorem:

$$\oint_C H \cdot dl = \int_S (\nabla \times H) \cdot ds$$

Hence,

$$\int_S (\nabla \times H) \cdot ds = \int_S J \cdot ds$$

Since the surface is arbitrary,

$$\boxed{\nabla \times H = J}$$

This is Maxwell's equation for magnetostatic field.

2. Gauss law for magnetic field

Magnetic monopoles do not exist.

Therefore, total magnetic flux through a closed surface is zero:

$$\oint_S B \cdot ds = 0$$

Using divergence theorem:

$$\oint_S B \cdot ds = \int_V \nabla \cdot B \, dv$$

Therefore,

$$\int_V \nabla \cdot B \, dv = 0$$

Since volume is arbitrary,

$$\boxed{\nabla \cdot B = 0}$$

This is Maxwell's equation which states that magnetic flux lines are continuous closed loops.

Final magnetostatic Maxwell equations

$$\boxed{\nabla \times H = J}$$

$$\nabla \cdot B = 0$$

Q4. State Maxwell's equations in time domain and integral form.

Answer:

Maxwell's equations completely describe electromagnetic fields.

They are written in differential and integral forms.

1. Gauss law for electric field

Differential form:

$$\nabla \cdot D = \rho_v$$

Integral form:

$$\oint_S D \cdot ds = \int_V \rho_v dv = Q_{enc}$$

This equation states that electric flux leaving a closed surface is equal to charge enclosed.

2. Gauss law for magnetic field

Differential form:

$$\nabla \cdot B = 0$$

Integral form:

$$\oint_S B \cdot ds = 0$$

This states that net magnetic flux through a closed surface is zero.

3. Faraday's law

A time-varying magnetic field produces an induced emf.

Integral form:

$$\oint_C E \cdot dl = -\frac{d}{dt} \int_S B \cdot ds$$

Using Stokes theorem:

$$\oint_C E \cdot dl = \int_S (\nabla \times E) \cdot ds$$

Therefore,

$$\int_S (\nabla \times E) \cdot ds = - \int_S \frac{\partial B}{\partial t} \cdot ds$$

Hence,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

4. Ampere-Maxwell law

A magnetic field is produced by conduction current and displacement current.

Integral form:

$$\oint_C H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

Using Stokes theorem:

$$\oint_C H \cdot dl = \int_S (\nabla \times H) \cdot ds$$

Thus,

$$\int_S (\nabla \times H) \cdot ds = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

Hence,

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Final Maxwell equations in time domain

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Q5. Write Maxwell's equations in phasor domain.

Answer:

For time-harmonic fields, the fields vary sinusoidally with time.

If the time variation is assumed as:

$$e^{j\omega t}$$

then differentiation with respect to time becomes:

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

Therefore, Maxwell's equations in phasor form are obtained from time-domain equations.

1. Gauss law for electric field

Time domain:

$$\nabla \cdot D = \rho_v$$

Phasor domain:

$$\boxed{\nabla \cdot D_s = \rho_{vs}}$$

Integral form:

$$\boxed{\oint_S D_s \cdot ds = \int_V \rho_{vs} dv}$$

2. Gauss law for magnetic field

Time domain:

$$\nabla \cdot B = 0$$

Phasor domain:

$$\boxed{\nabla \cdot B_s = 0}$$

Integral form:

$$\oint_S B_s \cdot ds = 0$$

3. Faraday's law

Time domain:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

In phasor domain:

$$\frac{\partial B}{\partial t} = j\omega B_s$$

Therefore,

$$\nabla \times E_s = -j\omega B_s$$

Integral form:

$$\oint_C E_s \cdot dl = -j\omega \int_S B_s \cdot ds$$

4. Ampere-Maxwell law

Time domain:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

In phasor domain:

$$\frac{\partial D}{\partial t} = j\omega D_s$$

Therefore,

$$\nabla \times H_s = J_s + j\omega D_s$$

Integral form:

$$\oint_C H_s \cdot dl = \int_S (J_s + j\omega D_s) \cdot ds$$

Final Maxwell equations in phasor domain

$$\nabla \cdot D_s = \rho_{vs}$$

$$\nabla \cdot B_s = 0$$

$$\nabla \times E_s = -j\omega B_s$$

$$\nabla \times H_s = J_s + j\omega D_s$$

Q6. Derive the relation between electric field intensity and electric scalar potential.

Answer:

Electric potential is defined as work done per unit charge in moving a charge from one point to another.

If a charge Q is moved from point A to point B , then:

$$V_{AB} = \frac{W}{Q}$$

Work done is:

$$dW = F \cdot dl$$

Since,

$$E = \frac{F}{Q}$$

we get:

$$dW = QE \cdot dl$$

For movement against the electric field:

$$dW = -QE \cdot dl$$

Integrating from A to B :

$$W = -Q \int_A^B E \cdot dl$$

Dividing by Q :

$$\frac{W}{Q} = - \int_A^B E \cdot dl$$

Therefore,

$$V_{AB} = - \int_A^B E \cdot dl$$

Also,

$$V_{AB} = V_B - V_A$$

For differential displacement:

$$dV = -E \cdot dl$$

In Cartesian coordinates:

$$dl = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

and

$$\mathbf{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

Therefore,

$$dV = -(E_x dx + E_y dy + E_z dz)$$

But,

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

Comparing both equations:

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

Therefore,

$$\boxed{\mathbf{E} = -\nabla V}$$

This is the relation between electric field intensity and electric scalar potential.

Q7. Derive the electric potential due to a point charge and due to system of point charges.

Answer:

Consider a point charge Q placed at origin.

Electric field intensity due to point charge is:

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

Potential difference is:

$$V_{AB} = - \int_A^B E \cdot dl$$

For radial path:

$$dl = dr \hat{a}_r$$

Therefore,

$$V_{AB} = - \int_A^B \frac{Q}{4\pi\epsilon_0 r^2} dr$$

$$V_{AB} = - \frac{Q}{4\pi\epsilon_0} \int_A^B \frac{dr}{r^2}$$

$$V_{AB} = - \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{r_A}^{r_B}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

If reference is at infinity:

$$r_A \rightarrow \infty$$

then,

$$\frac{1}{r_A} = 0$$

Hence potential at distance r is:

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$

Potential due to point charge not at origin

If point charge Q is located at position vector \mathbf{r}' , then potential at point \mathbf{r} is:

$$V(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|}$$

Potential due to N point charges

For N charges Q_1, Q_2, \dots, Q_N located at $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N$, potential at \mathbf{r} is obtained by superposition:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^N \frac{Q_k}{|\mathbf{r} - \mathbf{r}_k|}$$

Q8. Derive Poisson's equation and Laplace equation.

Answer:

From Gauss law:

$$\nabla \cdot \mathbf{D} = \rho_v$$

For a linear isotropic medium:

$$\mathbf{D} = \epsilon \mathbf{E}$$

Therefore,

$$\nabla \cdot (\epsilon \mathbf{E}) = \rho_v$$

If ϵ is constant:

$$\epsilon \nabla \cdot \mathbf{E} = \rho_v$$

We know that:

$$\mathbf{E} = -\nabla V$$

Substituting:

$$\epsilon \nabla \cdot (-\nabla V) = \rho_v$$

$$-\epsilon \nabla^2 V = \rho_v$$

Therefore,

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

This is called **Poisson's equation**.

Laplace equation

If there is no charge in the region, then:

$$\rho_v = 0$$

So Poisson's equation becomes:

$$\nabla^2 V = 0$$

Therefore,

$$\boxed{\nabla^2 V = 0}$$

This is called **Laplace equation**.

Physical meaning

Poisson's equation is used when charge density exists in the region.

Laplace equation is used in charge-free regions.

Q9. Derive the continuity equation of current.

Answer:

The continuity equation is based on the principle of conservation of charge.

It states that the rate of decrease of charge inside a volume is equal to the net outward current flowing through the closed surface.

Let current density be \mathbf{J} .

The total current leaving a closed surface is:

$$I_{out} = \oint_S \mathbf{J} \cdot d\mathbf{s}$$

The total charge inside volume V is:

$$Q_{in} = \int_V \rho_v dv$$

According to conservation of charge:

$$I_{out} = -\frac{dQ_{in}}{dt}$$

Therefore,

$$\oint_S \mathbf{J} \cdot d\mathbf{s} = -\frac{d}{dt} \int_V \rho_v dv$$

Using divergence theorem:

$$\oint_S \mathbf{J} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{J} dv$$

So,

$$\int_V \nabla \cdot J \, dv = - \int_V \frac{\partial \rho_v}{\partial t} dv$$

Therefore,

$$\int_V \left(\nabla \cdot J + \frac{\partial \rho_v}{\partial t} \right) dv = 0$$

Since the volume is arbitrary:

$$\boxed{\nabla \cdot J = - \frac{\partial \rho_v}{\partial t}}$$

or

$$\boxed{\nabla \cdot J + \frac{\partial \rho_v}{\partial t} = 0}$$

This is the continuity equation.

For steady current

For steady current:

$$\frac{\partial \rho_v}{\partial t} = 0$$

Therefore,

$$\boxed{\nabla \cdot J = 0}$$

This means current entering a volume is equal to current leaving it.

Q10. Derive relaxation time in a conducting medium.

Answer:

Relaxation time is the time taken by charge placed inside a material to decay to e^{-1} or 36.8% of its initial value.

From Ohm's law:

$$J = \sigma E$$

From Gauss law:

$$\nabla \cdot D = \rho_v$$

For a medium:

$$D = \epsilon E$$

Therefore,

$$\nabla \cdot (\epsilon E) = \rho_v$$

For constant ϵ :

$$\epsilon \nabla \cdot E = \rho_v$$

Thus,

$$\nabla \cdot E = \frac{\rho_v}{\epsilon}$$

Now,

$$\nabla \cdot J = \nabla \cdot (\sigma E)$$

For constant σ :

$$\nabla \cdot J = \sigma \nabla \cdot E$$

$$\nabla \cdot J = \frac{\sigma \rho_v}{\epsilon}$$

From continuity equation:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

Therefore,

$$\frac{\sigma \rho_v}{\epsilon} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} = -\frac{\sigma}{\epsilon} \rho_v$$

Separating variables:

$$\frac{d\rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} dt$$

Integrating:

$$\int \frac{d\rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} \int dt$$

$$\ln \rho_v = -\frac{\sigma}{\epsilon} t + C$$

At $t = 0$,

$$\rho_v = \rho_{v0}$$

Therefore,

$$C = \ln \rho_{v0}$$

So,

$$\ln \left(\frac{\rho_v}{\rho_{v0}} \right) = -\frac{\sigma}{\epsilon} t$$

Taking antilog:

$$\rho_v = \rho_{v0} e^{-\frac{\sigma}{\epsilon} t}$$

Let,

$$\tau = \frac{\epsilon}{\sigma}$$

Therefore,

$$\rho_v = \rho_{v0} e^{-t/\tau}$$

where,

$$\tau = \frac{\epsilon}{\sigma}$$

is called relaxation time.

Meaning

For a good conductor, σ is large, so τ is very small.

Thus charge disappears very quickly from inside conductor.

For a good dielectric, σ is very small, so τ is large.

Thus charge remains for long time.

Q11. Derive Joule's law in field form.

Answer:

Joule's law gives power dissipated in a conducting medium due to current flow.

From Ohm's law:

$$J = \sigma E$$

Also,

$$J = \frac{I}{A}$$

and electric field is:

$$E = \frac{V}{l}$$

Therefore,

$$\sigma E = \frac{I}{A}$$

$$\sigma \frac{V}{l} = \frac{I}{A}$$

$$\frac{V}{I} = \frac{l}{\sigma A}$$

But,

$$R = \frac{V}{I}$$

Therefore,

$$R = \frac{l}{\sigma A}$$

This is resistance in terms of conductivity.

Power dissipated

Power is work done per unit time.

$$P = \frac{W}{t}$$

Force on charge is:

$$F = QE$$

If charge moves through displacement l , then work done:

$$W = F \cdot l = QE \cdot l$$

Current is rate of flow of charge:

$$I = \frac{Q}{t}$$

So,

$$P = \frac{QE \cdot l}{t}$$

$$P = IE \cdot l$$

For volume distribution:

$$I = \rho_v v ds$$

and current density is:

$$J = \rho_v v$$

Hence power dissipated in volume is:

$$P = \int_V E \cdot J dv$$

This is Joule's law in field form.

Q12. Explain displacement current density and derive modified Ampere's law.

Answer:

Ampere's law for steady fields is:

$$\nabla \times H = J$$

Taking divergence on both sides:

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J$$

But divergence of curl is always zero:

$$\nabla \cdot (\nabla \times H) = 0$$

Therefore,

$$\nabla \cdot J = 0$$

This is valid only for steady current.

But for time-varying fields, continuity equation gives:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

So, Ampere's law must be modified.

From Gauss law:

$$\nabla \cdot D = \rho_v$$

Differentiating with respect to time:

$$\frac{\partial}{\partial t} (\nabla \cdot D) = \frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \frac{\partial D}{\partial t} = \frac{\partial \rho_v}{\partial t}$$

From continuity equation:

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

Therefore,

$$\nabla \cdot J = -\nabla \cdot \frac{\partial D}{\partial t}$$

$$\nabla \cdot \left(J + \frac{\partial D}{\partial t} \right) = 0$$

Hence Maxwell introduced displacement current density:

$$J_d = \frac{\partial D}{\partial t}$$

Therefore total current density is:

$$J + J_d$$

Modified Ampere's law becomes:

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

or

$$\nabla \times H = J + J_d$$

Integral form

$$\oint_C H \cdot dl = \int_S \left(J + \frac{\partial D}{\partial t} \right) \cdot ds$$

This is one of Maxwell's equations for time-varying fields.

Q13. Derive expression for displacement current in a parallel plate capacitor.

Answer:

Given a parallel plate capacitor with plate area A , separation d , dielectric permittivity ϵ , and applied voltage:

$$V = V_m \sin \omega t$$

Electric field between plates is:

$$E = \frac{V}{d}$$

Electric flux density:

$$D = \epsilon E$$

$$D = \epsilon \frac{V}{d}$$

Displacement current density is:

$$J_d = \frac{\partial D}{\partial t}$$

$$J_d = \frac{\partial}{\partial t} \left(\epsilon \frac{V}{d} \right)$$

Since ϵ and d are constants:

$$J_d = \frac{\epsilon}{d} \frac{\partial V}{\partial t}$$

For:

$$V = V_m \sin \omega t$$

$$\frac{\partial V}{\partial t} = V_m \omega \cos \omega t$$

Therefore,

$$J_d = \frac{\epsilon V_m \omega}{d} \cos \omega t$$

Total displacement current is:

$$I_d = J_d A$$

Hence,

$$I_d = \frac{\epsilon A V_m \omega}{d} \cos \omega t$$

For your note example

If:

$$A = 5 \text{ cm}^2, \quad d = 3 \text{ mm}, \quad V = 50 \sin 10^3 t, \quad \epsilon = 2\epsilon_0$$

then:

$$J_d = \frac{2\epsilon_0}{3 \times 10^{-3}} (50 \times 10^3 \cos 10^3 t)$$

and

$$I_d = J_d A$$

The answer in notes is:

$$I_d = 147.4 \cos 10^3 t \text{ nA}$$

Q14. State and derive Coulomb's law and electric field due to system of point charges.

Answer:

Coulomb's law gives the force between two point charges.

Let two point charges Q_1 and Q_2 be located at position vectors \mathbf{r}_1 and \mathbf{r}_2 .

The force on Q_2 due to Q_1 is:

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{a}_R$$

where,

$$R = r_2 - r_1$$

and

$$\hat{a}_R = \frac{r_2 - r_1}{|r_2 - r_1|}$$

Therefore,

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |r_2 - r_1|^2} \frac{r_2 - r_1}{|r_2 - r_1|}$$

$$F_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\epsilon_0 |r_2 - r_1|^3}$$

Electric field intensity

Electric field intensity is force per unit test charge.

$$E = \frac{F}{Q}$$

For a point charge Q :

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_R$$

or,

$$E = \frac{Q(r - r')}{4\pi\epsilon_0 |r - r'|^3}$$

Electric field due to N point charges

Using superposition principle:

$$E = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^N \frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}$$

This expression gives electric field at point \mathbf{r} due to N point charges.

Q15. Derive electric flux density due to point charge, line charge and sheet charge using Gauss law.

Answer:

Gauss law is:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enc}$$

1. Point charge

For point charge Q , choose spherical Gaussian surface of radius r .

Due to symmetry:

$$\mathbf{D} = D_r \hat{\mathbf{a}}_r$$

Surface element:

$$d\mathbf{s} = r^2 \sin \theta d\theta d\phi \hat{\mathbf{a}}_r$$

Then:

$$\oint_S D \cdot ds = Q$$

$$D_r \int_0^\pi \int_0^{2\pi} r^2 \sin \theta d\phi d\theta = Q$$

$$D_r r^2 (2\pi)(2) = Q$$

$$D_r 4\pi r^2 = Q$$

Therefore,

$$D = \frac{Q}{4\pi r^2} \hat{a}_r$$

Since $D = \epsilon_0 E$:

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

2. Infinite line charge

Let line charge density be ρ_l .

Choose cylindrical Gaussian surface of radius ρ and length l .

$$D = D_\rho \hat{a}_\rho$$

Surface element on curved surface:

$$ds = \rho d\phi dz \hat{a}_\rho$$

Gauss law:

$$\oint_S D \cdot ds = \rho_l l$$

$$D_{\rho} \int_0^{2\pi} \int_0^l \rho d\phi dz = \rho l$$

$$D_{\rho} \rho (2\pi) l = \rho l$$

Therefore,

$$D = \frac{\rho l}{2\pi \rho} \hat{a}_{\rho}$$

and

$$E = \frac{\rho l}{2\pi \epsilon \rho} \hat{a}_{\rho}$$

3. Infinite sheet charge

Let surface charge density be ρ_s .

Choose a pillbox Gaussian surface.

Flux leaves through top and bottom surfaces.

$$\oint_S D \cdot ds = \rho_s A$$

$$DA + DA = \rho_s A$$

$$2D = \rho_s$$

Therefore,

$$D = \frac{\rho_s}{2} \hat{a}_n$$

and

$$E = \frac{\rho_s}{2\epsilon} \hat{a}_n$$

Q16. Derive electric flux density due to uniformly charged sphere.

Answer:

Let a sphere of radius a have uniform volume charge density ρ_v .

Using Gauss law:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enc}$$

Due to spherical symmetry:

$$\mathbf{D} = D_r \hat{a}_r$$

Case 1: Inside the sphere ($r \leq a$)

Gaussian surface radius is r .

Surface area:

$$4\pi r^2$$

Charge enclosed:

$$Q_{enc} = \rho_v \left(\frac{4}{3} \pi r^3 \right)$$

Gauss law:

$$D_r(4\pi r^2) = \rho_v \frac{4}{3} \pi r^3$$

$$D_r = \frac{\rho_v r}{3}$$

Therefore,

$$D = \frac{\rho_v r}{3} \hat{a}_r \quad (r \leq a)$$

Case 2: Outside the sphere ($r \geq a$)

Gaussian surface radius is r .

Charge enclosed is total charge of sphere:

$$Q_{enc} = \rho_v \frac{4}{3} \pi a^3$$

Gauss law:

$$D_r (4\pi r^2) = \rho_v \frac{4}{3} \pi a^3$$

$$D_r = \frac{\rho_v a^3}{3r^2}$$

Therefore,

$$D = \frac{\rho_v a^3}{3r^2} \hat{a}_r \quad (r \geq a)$$

Q17. State and derive Biot-Savart law and Ampere's circuit law.

Answer:

1. Biot-Savart law

Biot-Savart law gives the magnetic field intensity produced by a differential current element.

A current element $I d\mathbf{l}$ produces differential magnetic field intensity $d\mathbf{H}$ at point P .

According to Biot-Savart law:

$$dH \propto \frac{I dl \sin \alpha}{R^2}$$

Therefore,

$$dH = \frac{I dl \sin \alpha}{4\pi R^2}$$

In vector form:

$$d\mathbf{H} = \frac{I d\mathbf{l} \times \hat{\mathbf{a}}_R}{4\pi R^2}$$

where:

- I = current
 - $d\mathbf{l}$ = differential length element
 - R = distance from current element to observation point
 - α = angle between $d\mathbf{l}$ and R
-

2. Ampere's circuit law

Ampere's circuit law states that the line integral of magnetic field intensity around a closed path is equal to total current enclosed.

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

For current density \mathbf{J} :

$$I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

Thus,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

Using Stokes theorem:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s}$$

Therefore,

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

Hence,

$$\nabla \times \mathbf{H} = \mathbf{J}$$

This is Maxwell's equation for magnetostatic field.

Q18. Derive magnetic field intensity due to an infinite line current using Ampere's law.

Answer:

Consider an infinite line current I along z -axis.

Due to symmetry, magnetic field intensity is circular around the conductor.

Thus,

$$\mathbf{H} = H_\phi \hat{\mathbf{a}}_\phi$$

Choose circular Amperian path of radius ρ .

Ampere's circuit law:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

For circular path:

$$d\mathbf{l} = \rho d\phi \hat{\mathbf{a}}_\phi$$

Therefore,

$$\int_0^{2\pi} H_\phi \hat{\mathbf{a}}_\phi \cdot \rho d\phi \hat{\mathbf{a}}_\phi = I$$

$$H_\phi \rho \int_0^{2\pi} d\phi = I$$

$$H_\phi \rho (2\pi) = I$$

Therefore,

$$H_\phi = \frac{I}{2\pi\rho}$$

Hence,

$$\mathbf{H} = \frac{I}{2\pi\rho} \hat{\mathbf{a}}_\phi$$

Magnetic flux density is:

$$\mathbf{B} = \mu \mathbf{H} = \frac{\mu I}{2\pi\rho} \hat{\mathbf{a}}_\phi$$

Q19. Derive magnetic field intensity due to infinite sheet current.

Answer:

Consider an infinite current sheet carrying surface current density K_y .

The magnetic field exists on both sides of the sheet.

Using Ampere's circuit law:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

Choose rectangular Amperian loop crossing the sheet.

From symmetry, magnetic field on both sides is equal and opposite.

For the rectangular path:

$$H(b) + H(b) = K_y b$$

$$2Hb = K_y b$$

Therefore,

$$H = \frac{K_y}{2}$$

Thus,

$$\boxed{H = \frac{K_y}{2} \hat{a}}$$

Direction is obtained using right-hand rule.

For the note case:

$$\boxed{H = \frac{K_y}{2} \hat{a}_y}$$

Magnetic flux density is:

$$\boxed{B = \mu H}$$

Q20. Explain magnetic vector potential and magnetic scalar potential.

Answer:

From Maxwell's equation for magnetic field:

$$\nabla \cdot B = 0$$

Also, from vector identity:

$$\nabla \cdot (\nabla \times A) = 0$$

Since divergence of curl is always zero, magnetic flux density B can be expressed as curl of a vector A .

Therefore,

$$\boxed{B = \nabla \times A}$$

where A is called **magnetic vector potential**.

Magnetic scalar potential

In current-free region:

$$J = 0$$

From Ampere's law:

$$\nabla \times H = J$$

Thus,

$$\nabla \times H = 0$$

If curl of a vector is zero, it can be expressed as negative gradient of scalar potential.

Therefore,

$$H = -\nabla V_m$$

where V_m is magnetic scalar potential.

Relation between B and A

Since:

$$B = \mu H$$

and

$$B = \nabla \times A$$

Magnetic flux through surface S is:

$$\psi = \int_S B \cdot ds$$

Substituting:

$$\psi = \int_S (\nabla \times A) \cdot ds$$

Using Stokes theorem:

$$\psi = \oint_C A \cdot dl$$

Thus magnetic flux can be obtained from magnetic vector potential.

Q21. Derive electric boundary conditions at dielectric-dielectric interface.

Answer:

Consider two dielectric media separated by an interface.

Let medium 1 have permittivity ϵ_1 and medium 2 have permittivity ϵ_2 .

Electric field in medium 1:

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

Electric field in medium 2:

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$

1. Tangential component of electric field

Using Maxwell's equation:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

Take a small rectangular loop across the boundary.

As height tends to zero, contribution from normal sides becomes zero.

Thus,

$$E_{1t}\Delta w - E_{2t}\Delta w = 0$$

$$E_{1t} = E_{2t}$$

Therefore,

$$\boxed{E_{1t} = E_{2t}}$$

Tangential electric field intensity is continuous across dielectric boundary.

Since:

$$D = \epsilon E$$

$$E = \frac{D}{\epsilon}$$

Thus,

$$\boxed{\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}}$$

Tangential electric flux density is generally discontinuous.

2. Normal component of electric flux density

Using Gauss law:

$$\oint_S D \cdot ds = Q_{enc}$$

Take a small pillbox across boundary.

Flux through side surface is zero.

Thus,

$$D_{2n}\Delta S - D_{1n}\Delta S = \rho_s\Delta S$$

$$D_{2n} - D_{1n} = \rho_s$$

Therefore,

$$\boxed{D_{2n} - D_{1n} = \rho_s}$$

If no free surface charge exists:

$$\rho_s = 0$$

then,

$$\boxed{D_{1n} = D_{2n}}$$

Since:

$$D = \epsilon E$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

or,

$$\boxed{\frac{E_{1n}}{E_{2n}} = \frac{\epsilon_2}{\epsilon_1}}$$

Q22. Derive law of refraction of electric field at dielectric interface.

Answer:

At dielectric-dielectric boundary, electric field changes direction due to change in permittivity.

Let E_1 and E_2 make angles θ_1 and θ_2 with normal to interface.

From boundary condition:

$$E_{1t} = E_{2t}$$

Therefore,

$$E_1 \sin \theta_1 = E_2 \sin \theta_2$$

Also, if no free surface charge:

$$D_{1n} = D_{2n}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\epsilon_1 E_1 \cos \theta_1 = \epsilon_2 E_2 \cos \theta_2$$

Divide first equation by second:

$$\frac{E_1 \sin \theta_1}{\epsilon_1 E_1 \cos \theta_1} = \frac{E_2 \sin \theta_2}{\epsilon_2 E_2 \cos \theta_2}$$
$$\frac{1}{\epsilon_1} \tan \theta_1 = \frac{1}{\epsilon_2} \tan \theta_2$$

Therefore,

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

Since:

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

This is law of refraction of electric field.

Q23. Derive electric boundary conditions at conductor-dielectric interface.

Answer:

Consider a conductor and dielectric interface.

Inside a perfect conductor:

$$E = 0$$

and

$$D = 0$$

1. Tangential component

Using:

$$\oint_C E \cdot dl = 0$$

Taking a small rectangular loop at boundary:

$$E_t \Delta w = 0$$

Therefore,

$$\boxed{E_t = 0}$$

Since:

$$D = \epsilon E$$

$$\boxed{D_t = 0}$$

Thus tangential electric field at conductor surface is zero.

2. Normal component

Using Gauss law:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enc}$$

Take pillbox at conductor-dielectric boundary.

Inside conductor:

$$\mathbf{D} = 0$$

Thus,

$$D_n \Delta S = \rho_s \Delta S$$

$$\boxed{D_n = \rho_s}$$

Since:

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\boxed{E_n = \frac{\rho_s}{\epsilon}}$$

Important notes

1. No electric field exists inside a perfect conductor:

$$\boxed{\mathbf{E} = 0}$$

2. Since:

$$\mathbf{E} = -\nabla V$$

inside conductor:

$$\nabla V = 0$$

So conductor is an equipotential body.

3. Electric field at conductor surface is normal to the surface.

4. At conductor surface:

$$D_n = \rho_s$$

and

$$E_t = 0$$

Q24. Derive magnetic boundary conditions at interface between two magnetic media.

Answer:

Consider two magnetic media separated by an interface.

Let permeability of medium 1 be μ_1 , and medium 2 be μ_2 .

Magnetic field intensity:

$$H = H_t + H_n$$

Magnetic flux density:

$$B = B_t + B_n$$

1. Tangential component of magnetic field intensity

Using Ampere's circuit law:

$$\oint_C H \cdot dl = I_{enc}$$

For small rectangular loop at boundary:

$$H_{2t}\Delta w - H_{1t}\Delta w = K\Delta w$$

where K is surface current density.

Thus,

$$H_{2t} - H_{1t} = K$$

In vector form:

$$\hat{a}_n \times (H_2 - H_1) = K$$

If no surface current exists:

$$K = 0$$

then,

$$H_{1t} = H_{2t}$$

Since:

$$B = \mu H$$

$$\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

Thus tangential B is generally discontinuous.

2. Normal component of magnetic flux density

Using Maxwell's equation:

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

For small pillbox:

$$B_{2n}\Delta S - B_{1n}\Delta S = 0$$

Therefore,

$$\boxed{B_{1n} = B_{2n}}$$

Since:

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

Thus normal component of \mathbf{H} is generally discontinuous.

Final magnetic boundary conditions

$$\boxed{H_{2t} - H_{1t} = K}$$

If $K = 0$:

$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{B_{1n} = B_{2n}}$$

Q25. Derive law of refraction of magnetic field.

Answer:

Let magnetic field in medium 1 and medium 2 make angles θ_1 and θ_2 with the normal to interface.

At boundary, if no surface current exists:

$$H_{1t} = H_{2t}$$

Thus,

$$H_1 \sin \theta_1 = H_2 \sin \theta_2$$

Also,

$$B_{1n} = B_{2n}$$

Since:

$$B = \mu H$$

$$\mu_1 H_1 \cos \theta_1 = \mu_2 H_2 \cos \theta_2$$

Dividing the tangential equation by normal equation:

$$\frac{H_1 \sin \theta_1}{\mu_1 H_1 \cos \theta_1} = \frac{H_2 \sin \theta_2}{\mu_2 H_2 \cos \theta_2}$$
$$\frac{1}{\mu_1} \tan \theta_1 = \frac{1}{\mu_2} \tan \theta_2$$

Therefore,

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}}$$

Since:

$$\mu = \mu_0 \mu_r$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_{r1}}{\mu_{r2}}}$$

This is the law of refraction of magnetic field.

Q26. Write a complete note on scalar and vector potentials.

Answer:

Potentials are used to simplify field calculations.

There are mainly two types:

1. Electric scalar potential
 2. Magnetic scalar and vector potential
-

1. Electric scalar potential

In electrostatic field:

$$\nabla \times \mathbf{E} = 0$$

Since curl of gradient is always zero:

$$\nabla \times (\nabla V) = 0$$

Therefore, electric field can be written as:

$$\boxed{\mathbf{E} = -\nabla V}$$

where V is electric scalar potential.

The potential difference between two points is:

$$\boxed{V_{AB} = - \int_A^B \mathbf{E} \cdot d\mathbf{l}}$$

Potential due to point charge is:

$$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$$

Potential due to N point charges is:

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^N \frac{Q_k}{|r - r_k|}$$

2. Magnetic scalar potential

In current-free region:

$$J = 0$$

From Ampere's law:

$$\nabla \times H = J$$

So,

$$\nabla \times H = 0$$

Hence,

$$H = -\nabla V_m$$

where V_m is magnetic scalar potential.

3. Magnetic vector potential

From Gauss law for magnetic field:

$$\nabla \cdot B = 0$$

Also:

$$\nabla \cdot (\nabla \times A) = 0$$

Therefore, magnetic flux density can be written as:

$$\boxed{B = \nabla \times A}$$

where A is magnetic vector potential.

Magnetic flux through surface is:

$$\psi = \int_S B \cdot ds$$

Substituting $B = \nabla \times A$:

$$\psi = \int_S (\nabla \times A) \cdot ds$$

Using Stokes theorem:

$$\boxed{\psi = \oint_C A \cdot dl}$$

Thus vector potential is useful for finding magnetic flux.

Q27. Explain Ampere's force law.

Answer:

Ampere's force law gives the force between current-carrying conductors or on a current element placed in magnetic field.

A current element placed in magnetic flux density B experiences force:

$$\boxed{dF = Idl \times B}$$

For a straight conductor of length l :

$$\boxed{F = Il \times B}$$

Magnitude:

$$\boxed{F = IlB \sin \theta}$$

where θ is angle between current direction and magnetic flux density.

Force between two long parallel currents

Let two long parallel conductors carry currents I_1 and I_2 , separated by distance d .

Magnetic field produced by conductor 1 at conductor 2:

$$H_1 = \frac{I_1}{2\pi d}$$

$$B_1 = \mu H_1 = \frac{\mu I_1}{2\pi d}$$

Force on conductor 2 per unit length:

$$\frac{F}{l} = I_2 B_1$$

$$\frac{F}{l} = I_2 \frac{\mu I_1}{2\pi d}$$

Therefore,

$$\boxed{\frac{F}{l} = \frac{\mu I_1 I_2}{2\pi d}}$$

If currents are in same direction, force is attractive.

If currents are in opposite direction, force is repulsive.

Q28. Write short note on differential operators used in EMTL.

Answer:

In EMTL, vector operators are used to represent field variation.

1. Gradient

Gradient of scalar V gives a vector.

Cartesian:

$$\nabla V = \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z$$

Cylindrical:

$$\nabla V = \frac{\partial V}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z$$

Spherical:

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi$$

2. Divergence

Divergence converts vector into scalar.

Cartesian:

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cylindrical:

$$\nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Spherical:

$$\nabla \cdot A = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

3. Curl

Curl gives circulation of vector field.

Cartesian:

$$\nabla \times A = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical:

$$\nabla \times A = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

Important identities

$$\nabla \times (\nabla V) = 0$$

$$\nabla \cdot (\nabla \times A) = 0$$

Q29. State and explain divergence theorem and Stokes theorem.

Answer:

1. Divergence theorem

Divergence theorem states that the total outward flux of a vector field through a closed surface is equal to the volume integral of divergence of that vector field.

$$\oint_S \mathbf{A} \cdot d\mathbf{s} = \int_V (\nabla \cdot \mathbf{A}) dv$$

where:

- S is closed surface
- V is volume enclosed by S

This theorem converts surface integral into volume integral.

It is used in deriving Gauss law point form:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enc}$$
$$\int_V \nabla \cdot \mathbf{D} dv = \int_V \rho_v dv$$

Therefore,

$$\nabla \cdot \mathbf{D} = \rho_v$$

2. Stokes theorem

Stokes theorem states that circulation of a vector field around a closed path is equal to surface integral of curl of the vector over the open surface bounded by the path.

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$$

It converts line integral into surface integral.

It is used in deriving Maxwell's equations such as:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

Using Stokes theorem:

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = 0$$

Therefore,

$$\nabla \times \mathbf{E} = 0$$

Similarly,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{enc}$$

gives:

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Q30. Write coordinate transformation relations between Cartesian, cylindrical and spherical systems.

Answer:

1. Cylindrical coordinate system

Cylindrical coordinates are:

$$(\rho, \phi, z)$$

where:

- ρ = radial distance from z -axis
- ϕ = azimuthal angle
- z = same as Cartesian z

Relations:

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

Vector transformation

From Cartesian to cylindrical:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

From cylindrical to Cartesian:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

2. Spherical coordinate system

Spherical coordinates are:

$$(r, \theta, \phi)$$

where:

- r = radius of sphere
- θ = angle with z -axis
- ϕ = azimuthal angle

Relations:

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

Also:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

Vector transformation

Cartesian to spherical:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

SUPER IMPORTANT FINAL REVISION LIST

For tomorrow's exam, revise these derivations first:

1. Maxwell equations in time domain
 2. Maxwell equations in phasor domain
 3. Derivation of displacement current density
 4. Continuity equation
 5. Relaxation time
 6. Joule's law
 7. Poisson and Laplace equations
 8. Electric scalar potential and $\mathbf{E} = -\nabla V$
 9. Magnetic vector potential $\mathbf{B} = \nabla \times \mathbf{A}$
 10. Electric boundary conditions
 11. Magnetic boundary conditions
 12. Law of refraction for electric field
 13. Law of refraction for magnetic field
 14. Gauss law applications
 15. Ampere's circuit law applications
 16. Biot-Savart law
 17. Coulomb's law
 18. Coordinate transformations
 19. Divergence theorem
 20. Stokes theorem
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