Below are **exam-ready 10-mark questions and answers** for **Unit-2: Plane Wave Propagation**, using only the topics and equations from your notes.

UNIT–II: PLANE WAVE PROPAGATION

10-Mark Descriptive Questions and Answers

Q1. Define uniform plane wave. Derive the wave equations for conducting medium from Maxwell’s equations.

Answer:

A **uniform plane wave** is an electromagnetic wave in which the electric field and magnetic field are uniform over any plane perpendicular to the direction of propagation. The fields vary only along the direction of propagation.

For a wave propagating in $+z$ -direction,

$$\vec{E} = E_x(z)\hat{a}_x$$

$$\vec{H} = H_y(z)\hat{a}_y$$

The wave direction is given by

$$\vec{E} \times \vec{H}$$

If

$$\vec{E} \parallel \hat{a}_x, \qquad \vec{H} \parallel \hat{a}_y$$

then

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

so the wave propagates in $+z$ -direction.

For a conducting medium,

$$\sigma \neq 0, \qquad \epsilon = \epsilon_0 \epsilon_r, \qquad \mu = \mu_0 \mu_r$$

Assume sinusoidal time variation:

$$e^{j\omega t}$$

The phasor Maxwell's equations are:

$$\nabla \cdot \vec{E}_s = 0$$

$$\nabla \cdot \vec{B}_s = 0$$

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

$$\nabla \times \vec{H}_s = (\sigma + j\omega\epsilon)\vec{E}_s$$

Taking curl on both sides of Faraday's law,

$$\nabla \times (\nabla \times \vec{E}_s) = \nabla \times (-j\omega\mu \vec{H}_s)$$

$$\nabla \times (\nabla \times \vec{E}_s) = -j\omega\mu(\nabla \times \vec{H}_s)$$

Using Ampere's law,

$$\nabla \times \vec{H}_s = (\sigma + j\omega\epsilon)\vec{E}_s$$

Therefore,

$$\nabla \times (\nabla \times \vec{E}_s) = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s$$

Using vector identity,

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

For charge-free medium,

$$\nabla \cdot \vec{E}_s = 0$$

Hence,

$$-\nabla^2 \vec{E}_s = -j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s$$

$$\nabla^2 \vec{E}_s = j\omega\mu(\sigma + j\omega\epsilon)\vec{E}_s$$

Let

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

Therefore,

$$\boxed{\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0}$$

Similarly, for magnetic field,

$$\boxed{\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0}$$

These are called **Helmholtz wave equations.**

The propagation constant is

$$\boxed{\gamma = \alpha + j\beta}$$

where

$$\alpha = \text{attenuation constant}$$

$$\beta = \text{phase constant}$$

Q2. Derive the expressions for attenuation constant α and phase constant β in a conducting medium.

Answer:

For a conducting medium,

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

Since

$$\gamma = \alpha + j\beta$$

we write

$$(\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

Expanding LHS,

$$(\alpha + j\beta)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

RHS becomes

$$j\omega\mu\sigma + j^2\omega^2\mu\epsilon$$

$$= -\omega^2\mu\epsilon + j\omega\mu\sigma$$

Therefore,

$$\alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\mu\epsilon + j\omega\mu\sigma$$

Comparing real and imaginary parts,

$$\boxed{\alpha^2 - \beta^2 = -\omega^2\mu\epsilon}$$

$$\boxed{2\alpha\beta = \omega\mu\sigma}$$

Now,

$$|\gamma^2| = \sqrt{(-\omega^2\mu\epsilon)^2 + (\omega\mu\sigma)^2}$$

$$|\gamma^2| = \omega\mu\sqrt{\omega^2\epsilon^2 + \sigma^2}$$

But,

$$|\gamma^2| = |(\alpha + j\beta)^2|$$

$$|\gamma^2| = \alpha^2 + \beta^2$$

Therefore,

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\omega^2\epsilon^2 + \sigma^2}$$

$$\alpha^2 + \beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}$$

Now adding,

$$(\alpha^2 + \beta^2) + (\alpha^2 - \beta^2)$$

$$2\alpha^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - \omega^2\mu\epsilon$$

$$2\alpha^2 = \omega^2\mu\epsilon\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]$$

$$\alpha^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]$$

Hence,

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}}$$

Similarly, subtracting,

$$(\alpha^2 + \beta^2) - (\alpha^2 - \beta^2)$$

$$2\beta^2 = \omega^2\mu\epsilon\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + \omega^2\mu\epsilon$$

$$\beta^2 = \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]$$

Hence,

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}}$$

These are the general expressions for a conducting or lossy dielectric medium.

Q3. Explain wave propagation in lossy dielectric medium and derive the electric and magnetic field equations.

Answer:

A **lossy dielectric** is a medium in which the electromagnetic wave loses power as it propagates because of finite conductivity.

For lossy dielectric,

$$\sigma \neq 0$$

$$\epsilon = \epsilon_0\epsilon_r$$

$$\mu = \mu_0\mu_r$$

The propagation constant is

$$\gamma = \alpha + j\beta$$

where

$$\alpha = \textbf{attenuation constant}$$

$$\beta = \textbf{phase constant}$$

For wave propagating in $+z$ -direction, the phasor electric field is

$$\vec{E}_s(z) = E_0 e^{-\gamma z}\hat{a}_x$$

Substitute

$$\gamma = \alpha + j\beta$$

$$\vec{E}_s(z) = E_0 e^{-(\alpha+j\beta)z}\hat{a}_x$$

$$\vec{E}_s(z) = E_0 e^{-\alpha z} e^{-j\beta z}\hat{a}_x$$

Including time factor $e^{j\omega t}$,

$$\vec{E}(z,t) = \mathrm{Re}\left[E_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]\hat{a}_x$$

$$\boxed{\vec{E}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_x}$$

The magnetic field is related to electric field by intrinsic impedance:

$$\eta = \frac{E_0}{H_0}$$

Therefore,

$$H_0 = \frac{E_0}{\eta}$$

Since η is complex in lossy medium,

$$\eta = |\eta| e^{j\theta_\eta}$$

Hence,

$$\frac{1}{\eta} = \frac{1}{|\eta|} e^{-j\theta_\eta}$$

Therefore,

$$\boxed{\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{a}_y}$$

Thus, in a lossy dielectric:

$$\boxed{\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{a}_y}$$

Important points:

1. The amplitude decreases as

$$e^{-\alpha z}$$

2. Electric and magnetic fields are not in phase.
3. E leads H by angle θ_η .
4. The wave travels in $+z$ -direction.

Q4. Derive the intrinsic impedance of a lossy dielectric medium.

Answer:

Intrinsic impedance is defined as the ratio of electric field intensity to magnetic field intensity.

$$\boxed{\eta = \frac{E_0}{H_0}}$$

For a wave propagating in $+z$ -direction,

$$\vec{E}_s = E_0 e^{-\gamma z} \hat{a}_x$$

$$\vec{H}_s = H_0 e^{-\gamma z} \hat{a}_y$$

Using Maxwell's equation,

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

For $\vec{E}_s = E_x(z)\hat{a}_x$,

$$\nabla \times \vec{E}_s = \frac{\partial E_x}{\partial z} \hat{a}_y$$

Since

$$E_x = E_0 e^{-\gamma z}$$

$$\frac{\partial E_x}{\partial z} = -\gamma E_0 e^{-\gamma z}$$

Therefore,

$$\nabla \times \vec{E}_s = -\gamma E_0 e^{-\gamma z} \hat{a}_y$$

From Maxwell equation,

$$-\gamma E_0 e^{-\gamma z} \hat{a}_y = -j\omega\mu H_0 e^{-\gamma z} \hat{a}_y$$

Cancel common terms,

$$\gamma E_0 = j\omega\mu H_0$$

$$\frac{E_0}{H_0} = \frac{j\omega\mu}{\gamma}$$

Hence,

$$\boxed{\eta = \frac{j\omega\mu}{\gamma}}$$

But,

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

Therefore,

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

Dividing numerator and denominator by $j\omega\epsilon$,

$$\eta = \sqrt{\frac{\mu}{\epsilon}\frac{1}{1 - j\frac{\sigma}{\omega\epsilon}}}$$

Therefore,

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}}}$$

Magnitude of intrinsic impedance is

$$\boxed{|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}}$$

Phase angle is

$$\boxed{\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

Therefore,

$$\boxed{\eta = |\eta| e^{j\theta_\eta}}$$

In lossy dielectric, η is complex, hence E and H are out of phase.

Q5. Explain plane wave propagation in lossless dielectric medium. Derive α , β , velocity, wavelength and intrinsic impedance.

Answer:

A **lossless dielectric** is a medium in which conductivity is zero.

$$\boxed{\sigma = 0}$$

Also,

$$\epsilon = \epsilon_0 \epsilon_r$$

$$\mu = \mu_0 \mu_r$$

The general expressions are:

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

For lossless dielectric,

$$\sigma = 0$$

Therefore,

$$\frac{\sigma}{\omega\epsilon} = 0$$

Attenuation constant

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+0} - 1\right]}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}(1-1)}$$

$$\boxed{\alpha = 0}$$

Hence, there is no attenuation.

Phase constant

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+0} + 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}(1+1)}$$

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}}$$

Velocity of wave

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega\sqrt{\mu\epsilon}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\epsilon}}}$$

Intrinsic impedance

General expression:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

For $\sigma = 0$,

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}}$$

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}}$$

Since η is real, E and H are in phase.

Wavelength

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}}$$

Since

$$\omega = 2\pi f$$

$$\lambda = \frac{2\pi}{2\pi f\sqrt{\mu\epsilon}}$$

$$\boxed{\lambda = \frac{1}{f\sqrt{\mu\epsilon}}}$$

Field equations

For wave in $+z$ -direction,

$$\boxed{\vec{E} = E_0 \cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H} = H_0 \cos(\omega t - \beta z)\hat{a}_y}$$

where

$$H_0 = \frac{E_0}{\eta}$$

Thus,

$$\boxed{\vec{H} = \frac{E_0}{\sqrt{\mu/\epsilon}} \cos(\omega t - \beta z)\hat{a}_y}$$

Q6. Explain plane wave propagation in free space and derive all important parameters.

Answer:

Free space is a special case of lossless dielectric.

For free space,

$$\sigma = 0$$

$$\epsilon = \epsilon_0$$

$$\mu = \mu_0$$

Since $\sigma = 0$, free space is lossless.

Attenuation constant

$$\boxed{\alpha = 0}$$

Therefore, wave does not attenuate in free space.

Phase constant

For lossless medium,

$$\beta = \omega\sqrt{\mu\epsilon}$$

For free space,

$$\boxed{\beta = \omega\sqrt{\mu_0\epsilon_0}}$$

Since

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

we get

$$\boxed{\beta = \frac{\omega}{c}}$$

Velocity

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\boxed{v = c = 3 \times 10^8 \text{ m/s}}$$

Intrinsic impedance

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\boxed{\eta_0 = 120\pi\ \Omega}$$

$$\boxed{\eta_0 \approx 377\ \Omega}$$

Wavelength

$$\lambda = \frac{2\pi}{\beta}$$

Since

$$\beta = \frac{\omega}{c}$$

$$\lambda = \frac{2\pi c}{\omega}$$

$$\omega = 2\pi f$$

$$\boxed{\lambda = \frac{c}{f}}$$

Field equations in free space

For wave propagating in $+z$ -direction,

$$\boxed{\vec{E} = E_0 \cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H} = H_0 \cos(\omega t - \beta z)\hat{a}_y}$$

where

$$\boxed{H_0 = \frac{E_0}{\eta_0}}$$

Thus,

$$\boxed{\vec{H} = \frac{E_0}{377}\cos(\omega t - \beta z)\hat{a}_y}$$

Q7. Explain wave propagation in good conductors. Derive α , β , η , velocity, wavelength and skin depth.

Answer:

A medium is called a **good conductor** if

$$\boxed{\sigma \gg \omega\epsilon}$$

or

$$\boxed{\frac{\sigma}{\omega\epsilon} \gg 1}$$

For good conductor, the conduction current density is much greater than the displacement current density.

Attenuation constant

General expression:

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

For good conductor,

$$\frac{\sigma}{\omega\epsilon} \gg 1$$

Therefore,

$$\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} \approx \frac{\sigma}{\omega\epsilon}$$

Hence,

$$\alpha \approx \omega\sqrt{\frac{\mu\epsilon}{2}\left[\frac{\sigma}{\omega\epsilon}\right]}$$

$$\alpha = \omega\sqrt{\frac{\mu\sigma}{2\omega}}$$

$$\boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Since

$$\omega = 2\pi f$$

$$\boxed{\alpha = \sqrt{\pi f\mu\sigma}}$$

Phase constant

General expression:

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

For good conductor,

$$\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} \approx \frac{\sigma}{\omega\epsilon}$$

Neglecting 1 ,

$$\beta \approx \omega\sqrt{\frac{\mu\epsilon}{2}\left[\frac{\sigma}{\omega\epsilon}\right]}$$

$$\boxed{\beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Therefore,

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

Intrinsic impedance

General expression:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

For good conductor,

$$\sigma \gg \omega\epsilon$$

So,

$$\sigma + j\omega\epsilon \approx \sigma$$

Therefore,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

Since

$$j = 1\angle 90^\circ$$

$$\sqrt{j} = 1\angle 45^\circ$$

Hence,

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ}$$

Therefore, in good conductor, E leads H by 45° .

Velocity of wave

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$\boxed{v = \sqrt{\frac{2\omega}{\mu\sigma}}}$$

Wavelength

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

Using $\omega = 2\pi f$,

$$\boxed{\lambda = \sqrt{\frac{4\pi}{f\mu\sigma}}}$$

Skin depth

Skin depth is the distance through which the wave amplitude decreases by a factor of e^{-1} .

The wave amplitude varies as

$$E = E_0 e^{-\alpha z}$$

At skin depth $z = \delta$,

$$E = E_0 e^{-1}$$

Therefore,

$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\alpha\delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

For good conductor,

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Therefore,

$$\boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

Field equations in good conductor

$$\boxed{\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\hat{a}_y}$$

Q8. Explain wave propagation in good dielectric and characterize conductors and dielectrics.

Answer:

A medium is characterized by the ratio

$$\boxed{\frac{\sigma}{\omega\epsilon}}$$

This ratio is called the **loss tangent.**

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$

It is the ratio of conduction current density to displacement current density.

Conduction current density:

$$J_c = \sigma E$$

Displacement current density:

$$J_d = j\omega\epsilon E$$

Therefore,

$$\tan\theta = \frac{|J_c|}{|J_d|}$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$

Characterization of medium

1. Good dielectric

If

$$\boxed{\sigma \ll \omega\epsilon}$$

then

$$\tan\theta \ll 1$$

The medium behaves as a good dielectric.

In good dielectric,

$$\frac{\sigma}{\omega\epsilon} \ll 1$$

The wave attenuation is very small.

Using general expressions,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

For a good dielectric,

$$\boxed{\alpha \text{ is very small}}$$

$$\boxed{\beta \approx \omega\sqrt{\mu\epsilon}}$$

$$\boxed{v \approx \frac{1}{\sqrt{\mu\epsilon}}}$$

$$\boxed{\eta \approx \sqrt{\frac{\mu}{\epsilon}}}$$

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2. Good conductor

If

$$\boxed{\sigma \gg \omega\epsilon}$$

then

$$\tan\theta \gg 1$$

The medium behaves as a good conductor.

For good conductor,

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ}$$

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3. Frequency dependence

A material can behave differently at different frequencies.

Since

$$\tan\theta = \frac{\sigma}{\omega\epsilon}$$

when frequency increases, ω increases and the loss tangent decreases.

Therefore:

- At low frequency, a medium may behave as a good conductor.
- At high frequency, the same medium may behave as a good dielectric.

Q9. Derive the relation between electric field and magnetic field for a uniform plane wave.

Answer:

Consider a uniform plane wave propagating in $+z$ -direction.

Let,

$$\vec{E}_s = E_0 e^{-\gamma z} \hat{a}_x$$

$$\vec{H}_s = H_0 e^{-\gamma z} \hat{a}_y$$

Using Maxwell's equation,

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

For

$$\vec{E}_s = E_x(z) \hat{a}_x$$

$$\nabla \times \vec{E}_s = \frac{\partial E_x}{\partial z} \hat{a}_y$$

Since

$$E_x = E_0 e^{-\gamma z}$$

$$\frac{\partial E_x}{\partial z} = -\gamma E_0 e^{-\gamma z}$$

Therefore,

$$-\gamma E_0 e^{-\gamma z} \hat{a}_y = -j\omega\mu H_0 e^{-\gamma z} \hat{a}_y$$

Canceling common terms,

$$\gamma E_0 = j\omega\mu H_0$$

$$\frac{E_0}{H_0} = \frac{j\omega\mu}{\gamma}$$

Thus,

$$\boxed{\eta = \frac{E_0}{H_0}}$$

$$\boxed{\eta = \frac{j\omega\mu}{\gamma}}$$

Also,

$$\boxed{H_0 = \frac{E_0}{\eta}}$$

Field equations in terms of η

If

$$\eta = |\eta| e^{j\theta_\eta}$$

then electric field is

$$\boxed{\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_x}$$

Magnetic field is

$$\boxed{\vec{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \hat{a}_y}$$

Similarly, if magnetic field is given,

$$\boxed{\vec{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_y}$$

then

$$\vec{E} = \eta \vec{H}$$

$$\boxed{\vec{E} = |\eta| H_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_\eta) \hat{a}_x}$$

Therefore:

$$\boxed{E \text{ leads } H \text{ by } \theta_\eta}$$

$$\boxed{H \text{ lags } E \text{ by } \theta_\eta}$$

Q10. Explain sinusoidal uniform plane waves and write the complete field equations for waves travelling in $+z$ and $-z$ directions.

Answer:

For sinusoidal time variation,

$$e^{j\omega t}$$

The general solution of wave equation for electric field is

$$\boxed{E_x(z) = E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z}}$$

where

$$E_0^+ e^{-\gamma z}$$

represents wave travelling in $+z$ -direction, and

$$E_0^- e^{+\gamma z}$$

represents wave travelling in $-z$ -direction.

Wave travelling in $+z$ -direction

For $+z$ -direction,

$$E_x(z) = E_0 e^{-\gamma z}$$

$$\gamma = \alpha + j\beta$$

$$E_x(z) = E_0 e^{-\alpha z} e^{-j\beta z}$$

Including time factor,

$$E_x(z,t) = \mathrm{Re}\left[E_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]$$

$$\boxed{\vec{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{a}_y}$$

Wave travelling in $-z$ -direction

For $-z$ -direction,

$$E_x(z) = E_0 e^{+\gamma z}$$

$$E_x(z) = E_0 e^{+\alpha z} e^{+j\beta z}$$

With time factor,

$$E_x(z,t) = \mathrm{Re}\left[E_0 e^{+\alpha z} e^{j\beta z} e^{j\omega t}\right]$$

$$\boxed{\vec{E}(z,t) = E_0 e^{+\alpha z} \cos(\omega t + \beta z)\hat{a}_x}$$

For $-z$ -direction, magnetic field direction changes so that

$$\vec{E} \times \vec{H} = -\hat{a}_z$$

Therefore,

$$\boxed{\vec{H}(z,t) = -\frac{E_0}{|\eta|} e^{+\alpha z} \cos(\omega t + \beta z - \theta_\eta)\hat{a}_y}$$

Q11. Define skin depth and derive its expression for a conducting medium.

Answer:

When an electromagnetic wave travels through a conducting medium, its amplitude decreases exponentially.

The electric field is

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x$$

The amplitude part is

$$E_0 e^{-\alpha z}$$

Skin depth is defined as the distance through which the wave amplitude decreases to e^{-1} times its initial value.

Let skin depth be δ .

At

$$z = \delta$$

the amplitude becomes

$$E_0 e^{-1}$$

Therefore,

$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

Canceling E_0 ,

$$e^{-\alpha\delta} = e^{-1}$$

Therefore,

$$\alpha\delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

For a good conductor,

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Therefore,

$$\delta = \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$\boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

Since

$$\omega = 2\pi f$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}}$$

Thus, skin depth decreases when frequency, permeability or conductivity increases.

Q12. Explain polarization of uniform plane waves. Derive the general equation of polarization ellipse.

Answer:

Polarization is defined as the time-varying behaviour of the electric field strength at a fixed point in space.

Depending on the direction and variation of electric field, polarization is classified as:

$$\boxed{\text{Linear polarization}}$$

$$\boxed{\text{Circular polarization}}$$

$$\boxed{\text{Elliptical polarization}}$$

Let the electric field have two perpendicular components:

$$E_x = E_1 \cos \omega t$$

$$E_y = E_2 \cos(\omega t + \delta)$$

Using trigonometric identity,

$$\cos(\omega t + \delta) = \cos \omega t \cos \delta - \sin \omega t \sin \delta$$

Therefore,

$$E_y = E_2 \left[\cos \omega t \cos \delta - \sin \omega t \sin \delta\right]$$

Since

$$E_x = E_1 \cos \omega t$$

$$\cos \omega t = \frac{E_x}{E_1}$$

Also,

$$\sin \omega t = \sqrt{1 - \cos^2 \omega t}$$

$$\sin \omega t = \sqrt{1 - \left(\frac{E_x}{E_1}\right)^2}$$

Substitute,

$$E_y = E_2\left[\frac{E_x}{E_1}\cos\delta - \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta\right]$$

Divide by E_2 ,

$$\frac{E_y}{E_2} = \frac{E_x}{E_1}\cos\delta - \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta$$

Rearrange,

$$\frac{E_x}{E_1}\cos\delta - \frac{E_y}{E_2} = \sqrt{1-\left(\frac{E_x}{E_1}\right)^2}\sin\delta$$

Squaring on both sides,

$$\left(\frac{E_x}{E_1}\cos\delta - \frac{E_y}{E_2}\right)^2 = \left[1-\left(\frac{E_x}{E_1}\right)^2\right]\sin^2\delta$$

Expanding,

$$\frac{E_x^2}{E_1^2}\cos^2\delta - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta - \frac{E_x^2}{E_1^2}\sin^2\delta$$

Bringing terms together,

$$\frac{E_x^2}{E_1^2}(\cos^2\delta + \sin^2\delta) - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta$$

Since,

$$\cos^2\delta + \sin^2\delta = 1$$

Therefore,

$$\boxed{\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta}$$

This is the general equation of polarization ellipse.

Q13. Explain linear, circular and elliptical polarization with conditions.

Answer:

The general polarization equation is

$$\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta$$

where,

$$E_x = E_1\cos\omega t$$

$$E_y = E_2\cos(\omega t + \delta)$$

---

1. Linear polarization

If

$$\delta = 0$$

then

$$\sin\delta = 0$$

$$\cos\delta = 1$$

Substitute in general equation,

$$\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2} + \frac{E_y^2}{E_2^2} = 0$$

This can be written as

$$\left(\frac{E_x}{E_1} - \frac{E_y}{E_2}\right)^2 = 0$$

Therefore,

$$\frac{E_x}{E_1} = \frac{E_y}{E_2}$$

$$\boxed{E_y = \frac{E_2}{E_1}E_x}$$

This is the equation of a straight line.

Hence, the wave is linearly polarized.

---

2. Circular polarization

If

$$E_1 = E_2 = E_0$$

and

$$\delta = \pm\frac{\pi}{2}$$

then

$$\cos\delta = 0$$

$$\sin^2\delta = 1$$

The general equation becomes

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1$$

$$\boxed{E_x^2 + E_y^2 = E_0^2}$$

This is the equation of a circle.

Hence, the wave is circularly polarized.

Depending on the sign of phase difference, it may be right circularly polarized or left circularly polarized.

---

3. Elliptical polarization

If

$$E_1 \neq E_2$$

and

$$\delta \neq 0$$

then the general equation represents an ellipse:

$$\boxed{\frac{E_x^2}{E_1^2} - \frac{2E_xE_y}{E_1E_2}\cos\delta + \frac{E_y^2}{E_2^2} = \sin^2\delta}$$

Hence, the wave is elliptically polarized.

Special case:

If

$$\delta = \pm\frac{\pi}{2}$$

and

$$E_1 \neq E_2$$

then

$$\boxed{\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1}$$

This is also elliptical polarization.

NUMERICAL PROBLEMS

Similar to the sums in your notes

Problem 1. A uniform plane wave propagates in a medium and is given by

$$\vec{E} = 2e^{-\alpha z} \sin(10^8 t - \beta z)\hat{a}_y \text{ V/m}$$

The medium is characterized by

$$\epsilon_r = 1, \qquad \mu_r = 20, \qquad \sigma = 3 \text{ S/m}$$

Find:

1. α
2. β
3. $\vec{H}$

Solution:

Given,

$$\omega = 10^8 \text{ rad/s}$$

$$\epsilon = \epsilon_0 \epsilon_r = \frac{10^{-9}}{36\pi} \times 1$$

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 20$$

Loss tangent:

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{10^8 \times \frac{10^{-9}}{36\pi}}$$

$$\frac{\sigma}{\omega\epsilon} = 3392.9$$

Since,

$$\frac{\sigma}{\omega\epsilon} \gg 1$$

the medium is a good conductor.

For good conductor,

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{10^8 \times 20 \times 4\pi \times 10^{-7} \times 3}{2}}$$

$$\boxed{\alpha = \beta = 61.39 \text{ Np/m}}$$

The intrinsic impedance is

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$|\eta| = \sqrt{\frac{10^8 \times 20 \times 4\pi \times 10^{-7}}{3}}$$

$$|\eta| = 28.92 \ \Omega$$

Therefore,

$$H_0 = \frac{E_0}{|\eta|}$$

$$H_0 = \frac{2}{28.92}$$

$$H_0 = 0.069 \text{ A/m}$$

Since

$$\vec{E} \parallel \hat{a}_y$$

and wave travels in $+z$ -direction, $\vec{H}$ must be in $-\hat{a}_x$ -direction.

Also, H lags E by 45° .

$$\boxed{\vec{H} = -0.069e^{-61.39z} \sin(10^8 t - 61.39z - 45^\circ)\hat{a}_x \text{ A/m}}$$

Problem 2. A plane wave of amplitude 100 V/m and frequency 300 MHz travels in a lossless medium having

$$\mu_r = 1, \qquad \epsilon_r = 9, \qquad \sigma = 0$$

Write the complete time-domain equations for $\vec{E}$ and $\vec{H}$.

Solution:

Given,

$$E_0 = 100 \text{ V/m}$$

$$f = 300 \text{ MHz} = 300 \times 10^6 \text{ Hz}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi(300 \times 10^6)$$

$$\omega = 1.884 \times 10^9 \text{ rad/s}$$

For lossless dielectric,

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\epsilon}$$

Also,

$$v = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$v = \frac{3 \times 10^8}{\sqrt{1 \times 9}}$$

$$v = 1 \times 10^8 \text{ m/s}$$

$$\beta = \frac{\omega}{v}$$

$$\beta = \frac{1.884 \times 10^9}{1 \times 10^8}$$

$$\boxed{\beta = 18.84 \text{ rad/m}}$$

Intrinsic impedance:

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta = 377\sqrt{\frac{1}{9}}$$

$$\eta = 125.66\ \Omega$$

Now,

$$H_0 = \frac{E_0}{\eta}$$

$$H_0 = \frac{100}{125.66}$$

$$H_0 = 0.796\ \mathrm{A/m}$$

Therefore,

$$\boxed{\vec{E} = 100\cos(1.884 \times 10^9 t - 18.84z)\hat{a}_x\ \mathrm{V/m}}$$

$$\boxed{\vec{H} = 0.796\cos(1.884 \times 10^9 t - 18.84z)\hat{a}_y\ \mathrm{A/m}}$$

Problem 3. A plane wave propagates in a medium having

$$\epsilon_r = 8, \qquad \mu_r = 2$$

The electric field is

$$\vec{E} = 0.5e^{-z/3}\sin(10^8 t - \beta z)\hat{a}_x\ \mathrm{V/m}$$

Find:

1. β
2. Loss tangent
3. Intrinsic impedance
4. Wave velocity
5. Magnetic field

Solution:

Given,

$$E_0 = 0.5$$

$$\alpha = \frac{1}{3}\ \mathrm{Np/m}$$

$$\omega = 10^8\ \mathrm{rad/s}$$

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 2$$

$$\mu = 2.513 \times 10^{-6}\ \mathrm{H/m}$$

$$\epsilon = \epsilon_0 \epsilon_r = \frac{10^{-9}}{36\pi} \times 8$$

$$\epsilon = 7.073 \times 10^{-11}\ \mathrm{F/m}$$

Use

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

Let

$$x = \frac{\sigma}{\omega\epsilon}$$

Then,

$$\alpha^2 = \omega^2 \frac{\mu\epsilon}{2}\left[\sqrt{1 + x^2} - 1\right]$$

Substituting values gives,

$$x = 0.5151$$

Therefore,

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon} = 0.5151}$$

Now,

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+x^2}+1\right]}$$

$$\boxed{\beta = 1.374\ \text{rad/m}}$$

Magnitude of intrinsic impedance:

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+x^2\right]^{1/4}}$$

$$|\eta| = 177.7\ \Omega$$

Phase angle:

$$\theta_\eta = \frac{1}{2}\tan^{-1} x$$

$$\theta_\eta = \frac{1}{2}\tan^{-1}(0.5151)$$

$$\boxed{\theta_\eta = 13.63^\circ}$$

Therefore,

$$\boxed{\eta = 177.7\angle 13.63^\circ\ \Omega}$$

Wave velocity:

$$v = \frac{\omega}{\beta}$$

$$v = \frac{10^8}{1.374}$$

$$\boxed{v = 72.78\times 10^6\ \text{m/s}}$$

Now,

$$H_0 = \frac{E_0}{|\eta|}$$

$$H_0 = \frac{0.5}{177.7}$$

$$H_0 = 2.813 \times 10^{-3} \text{ A/m}$$

Since E is in $\hat{a}_x$ -direction and wave travels in $+z$ -direction, H is in $\hat{a}_y$ -direction.

$$\boxed{\vec{H} = 2.813 \times 10^{-3} e^{-z/3} \sin(10^8 t - 1.374z - 13.63^\circ)\hat{a}_y \text{ A/m}}$$

Problem 4. Calculate the intrinsic impedance, propagation constant, skin depth and wave velocity for copper at

$$f = 100 \text{ MHz}$$

Given,

$$\sigma = 58 \times 10^6 \text{ S/m}$$

$$\mu_r = 1$$

Solution:

Given,

$$f = 100 \times 10^6 \text{ Hz}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 100 \times 10^6$$

$$\sigma = 58 \times 10^6 \text{ S/m}$$

$$\mu = \mu_0 = 4\pi \times 10^{-7}$$

Since copper is a good conductor,

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

$$\alpha = \beta = \sqrt{\pi(100 \times 10^6)(4\pi \times 10^{-7})(58 \times 10^6)}$$

$$\boxed{\alpha = \beta = 1.51 \times 10^5 \text{ m}^{-1}}$$

Propagation constant:

$$\gamma = \alpha + j\beta$$

$$\boxed{\gamma = 1.51 \times 10^5 + j1.51 \times 10^5 \text{ m}^{-1}}$$

Magnitude form:

$$|\gamma| = \sqrt{\alpha^2 + \beta^2}$$

$$|\gamma| = 2.14 \times 10^5$$

$$\boxed{\gamma = 2.14 \times 10^5 \angle 45^\circ \text{ m}^{-1}}$$

Intrinsic impedance:

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$|\eta| = \sqrt{\frac{2\pi(100 \times 10^6)(4\pi \times 10^{-7})}{58 \times 10^6}}$$

$$\boxed{|\eta| = 3.69 \times 10^{-3}\ \Omega}$$

$$\boxed{\eta = 3.69 \times 10^{-3}\angle 45^\circ\ \Omega}$$

Skin depth:

$$\delta = \frac{1}{\alpha}$$

$$\delta = \frac{1}{1.51 \times 10^5}$$

$$\boxed{\delta = 6.6 \times 10^{-6}\ \text{m}}$$

$$\boxed{\delta = 6.6\ \mu\text{m}}$$

Wave velocity:

$$v = \frac{\omega}{\beta}$$

$$v = \frac{2\pi(100 \times 10^6)}{1.51 \times 10^5}$$

$$\boxed{v = 4.16 \times 10^3\ \text{m/s}}$$

Problem 5. Determine the polarization of the following waves.

a)

$$\vec{E} = 4e^{-0.25z}\cos(\omega t - \beta z)\hat{a}_x + 3e^{-0.25z}\sin(\omega t - \beta z)\hat{a}_y$$

b)

$$\vec{H} = H_0 e^{-\beta z}\hat{a}_x - 2H_0 e^{-\beta z}\hat{a}_y$$

c)

$$\vec{E} = \sin(\omega t - \beta z)\hat{a}_x + \sin\left(\omega t - \beta z - \frac{\pi}{2}\right)\hat{a}_y$$

Solution:

a)

The components are:

$$E_x = 4e^{-0.25z}\cos(\omega t - \beta z)$$

$$E_y = 3e^{-0.25z}\sin(\omega t - \beta z)$$

Since

$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$$

the phase difference is

$$\delta = \frac{\pi}{2}$$

Also,

$$E_1 \neq E_2$$

because

$$4 \neq 3$$

Therefore,

$$\boxed{\text{Elliptical polarization}}$$

---

b)

The magnetic field has two components with no phase difference.

$$H_x = H_0 e^{-\beta z}$$

$$H_y = -2H_0 e^{-\beta z}$$

The phase difference is

$$\delta = 0$$

Therefore, the field traces a straight line.

$$\boxed{\text{Linear polarization}}$$

---

c)

The components are:

$$E_x = \sin(\omega t - \beta z)$$

$$E_y = \sin\left(\omega t - \beta z - \frac{\pi}{2}\right)$$

The amplitudes are equal:

$$E_1 = E_2 = 1$$

The phase difference is

$$\delta = \frac{\pi}{2}$$

Therefore,

$$\boxed{\text{Circular polarization}}$$

Quick Formula Sheet for Last-Minute Revision

$$\boxed{\gamma = \alpha + j\beta}$$

$$\boxed{\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$

$$\boxed{v = \frac{\omega}{\beta}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

$$\boxed{\delta = \frac{1}{\alpha}}$$

For lossless dielectric:

$$\boxed{\alpha = 0}$$

$$\boxed{\beta = \omega\sqrt{\mu\epsilon}}$$

$$\boxed{v = \frac{1}{\sqrt{\mu\epsilon}}}$$

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}}$$

For good conductor:

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}}$$

$$\boxed{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ}$$

$$\boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

$$\boxed{\vec{E} = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_x}$$

$$\boxed{\vec{H} = \frac{E_0}{|\eta|}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\hat{a}_y}$$

Made for Anand Sagar by Claude