UNIT–III EMTL: 10-Mark Questions & Answers

Plane Waves at Media Interface

Notation used exactly like your notes:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}, \qquad \beta = \omega\sqrt{\mu\varepsilon}, \qquad \gamma = \alpha + j\beta$$

For lossless dielectric:

$$\alpha = 0, \qquad \gamma = j\beta$$

Time factor is suppressed in phasor form. Actual time-domain field is obtained by:

$$\mathbf{E}(t) = \Re\{\mathbf{E}e^{j\omega t}\}$$

Q1. Derive reflection and transmission coefficients for normal incidence at a perfect dielectric boundary.

Answer:

Consider a plane wave normally incident from medium 1 to medium 2 at the boundary $z = 0$.

Medium 1 is for $z < 0$, medium 2 is for $z > 0$.

Let the incident, reflected and transmitted electric fields be:

$$\mathbf{E}_i = E_{i0} e^{-j\beta_1 z} \hat{a}_x$$

$$\mathbf{E}_r = E_{r0} e^{+j\beta_1 z} \hat{a}_x$$

$$\mathbf{E}_t = E_{t0} e^{-j\beta_2 z} \hat{a}_x$$

The corresponding magnetic fields are:

$$\mathbf{H}_i = \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z} \hat{a}_y$$

$$\mathbf{H}_r = -\frac{E_{r0}}{\eta_1} e^{+j\beta_1 z} \hat{a}_y$$

$$\mathbf{H}_t = \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z} \hat{a}_y$$

At $z = 0$, tangential electric field must be continuous:

$$E_{i0} + E_{r0} = E_{t0}$$

Also tangential magnetic field must be continuous:

$$H_{i0} + H_{r0} = H_{t0}$$

Since reflected magnetic field is negative in direction:

$$\frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$$

Define reflection coefficient:

$$\Gamma = \frac{E_{r0}}{E_{i0}}$$

and transmission coefficient:

$$\tau = \frac{E_{t0}}{E_{i0}}$$

From electric field boundary condition:

$$1 + \Gamma = \tau$$

From magnetic field boundary condition:

$$\frac{1-\Gamma}{\eta_1} = \frac{\tau}{\eta_2}$$

Substitute $\tau = 1 + \Gamma$:

$$\frac{1-\Gamma}{\eta_1} = \frac{1+\Gamma}{\eta_2}$$

$$\eta_2(1-\Gamma) = \eta_1(1+\Gamma)$$

$$\eta_2 - \eta_2\Gamma = \eta_1 + \eta_1\Gamma$$

$$\eta_2 - \eta_1 = \Gamma(\eta_1 + \eta_2)$$

Therefore,

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

Now,

$$\tau = 1 + \Gamma$$

$$\tau = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{\eta_2 + \eta_1 + \eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\boxed{\tau = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

Hence, for normal incidence at a perfect dielectric boundary:

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\boxed{\tau = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

Q2. Explain normal incidence on a perfect conductor and derive the standing wave equations.

Answer:

Consider a uniform plane wave normally incident on a perfect conductor placed at $z = 0$. Medium 1 is a perfect dielectric, and medium 2 is a perfect conductor.

For a perfect conductor:

$$\sigma \to \infty$$

The intrinsic impedance of a conductor is:

$$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

As:

$$\sigma \to \infty$$

$$\eta_2 \to 0$$

For normal incidence, reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Substitute $\eta_2 = 0$:

$$\Gamma = \frac{0 - \eta_1}{0 + \eta_1}$$

$$\boxed{\Gamma = -1}$$

Transmission coefficient is:

$$\tau = 1 + \Gamma$$

$$\tau = 1 - 1 = 0$$

$$\boxed{\tau = 0}$$

Therefore, the wave is completely reflected and no wave is transmitted into the perfect conductor.

Let the incident electric field be:

$$\mathbf{E}_i = E_{i0} e^{-j\beta z} \hat{a}_x$$

The reflected electric field is:

$$\mathbf{E}_r = E_{r0} e^{+j\beta z} \hat{a}_x$$

Since:

$$\Gamma = -1$$

$$E_{r0} = -E_{i0}$$

Therefore:

$$\mathbf{E}_r = -E_{i0} e^{+j\beta z} \hat{a}_x$$

The total electric field in medium 1 is:

$$\mathbf{E}_s = \mathbf{E}_i + \mathbf{E}_r$$

$$\mathbf{E}_s = E_{i0} e^{-j\beta z} \hat{a}_x - E_{i0} e^{+j\beta z} \hat{a}_x$$

$$\mathbf{E}_s = E_{i0} \left(e^{-j\beta z} - e^{+j\beta z} \right) \hat{a}_x$$

Using:

$$e^{-j\theta} - e^{j\theta} = -2j \sin\theta$$

$$\boxed{\mathbf{E}_s = -2jE_{i0} \sin\beta z \, \hat{a}_x}$$

In time domain:

$$\boxed{\mathbf{E}_s = 2E_{i0} \sin\beta z \sin\omega t \, \hat{a}_x}$$

Similarly, the magnetic fields are:

$$\mathbf{H}_i = \frac{E_{i0}}{\eta_1} e^{-j\beta z} \hat{a}_y$$

For reflected wave:

$$\mathbf{H}_r = -\frac{E_{r0}}{\eta_1} e^{+j\beta z} \hat{a}_y$$

Since $E_{r0} = -E_{i0}$,

$$\mathbf{H}_r = \frac{E_{i0}}{\eta_1} e^{+j\beta z} \hat{a}_y$$

Total magnetic field:

$$\mathbf{H}_s = \frac{E_{i0}}{\eta_1} \left(e^{-j\beta z} + e^{+j\beta z} \right) \hat{a}_y$$

Using:

$$e^{-j\theta} + e^{j\theta} = 2\cos\theta$$

$$\boxed{\mathbf{H}_s = 2H_{i0} \cos \beta z \; \hat{a}_y}$$

In time domain:

$$\boxed{\mathbf{H}_s = 2H_{i0} \cos \beta z \cos \omega t \; \hat{a}_y}$$

Thus, incident and reflected waves combine to form a standing wave.

At the conductor surface $z = 0$:

$$\mathbf{E}_s = 0$$

and

$$\mathbf{H}_s = 2H_{i0} \cos \omega t \; \hat{a}_y$$

Hence, the tangential electric field is zero at a perfect conductor surface.

Q3. Derive standing wave ratio for normal incidence at a dielectric-dielectric boundary.

Answer:

When a plane wave is normally incident from one lossless dielectric medium to another, part of the wave is reflected and part is transmitted.

The incident and reflected electric fields in medium 1 are:

$$\mathbf{E}_i = E_{i0} e^{-j\beta_1 z} \hat{a}_x$$

$$\mathbf{E}_r = E_{r0} e^{+j\beta_1 z} \hat{a}_x$$

Total electric field in medium 1 is:

$$\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r$$

$$\mathbf{E}_1 = E_{i0} e^{-j\beta_1 z} \hat{a}_x + E_{r0} e^{+j\beta_1 z} \hat{a}_x$$

Using reflection coefficient:

$$\Gamma = \frac{E_{r0}}{E_{i0}}$$

$$\mathbf{E}_1 = E_{i0} e^{-j\beta_1 z} \left[1 + \Gamma e^{2j\beta_1 z}\right] \hat{a}_x$$

The magnitude is:

$$|\mathbf{E}_1| = E_{i0} \left|1 + \Gamma e^{2j\beta_1 z}\right|$$

Maximum value occurs when reflected and incident fields are in phase:

$$|\mathbf{E}|_{\max} = E_{i0}(1 + |\Gamma|)$$

Minimum value occurs when they are out of phase:

$$|\mathbf{E}|_{\min} = E_{i0}(1 - |\Gamma|)$$

Standing wave ratio is defined as:

$$S = \frac{|\mathbf{E}|_{\max}}{|\mathbf{E}|_{\min}}$$

Therefore:

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

Also,

$$\boxed{|\Gamma| = \frac{S - 1}{S + 1}}$$

For normal incidence:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

If $\eta_2 > \eta_1$, then $\Gamma > 0$.
If $\eta_2 < \eta_1$, then $\Gamma < 0$.

The positions of maxima and minima depend on the sign of Γ .

For $\Gamma > 0$:

$$z_{\max} = \frac{n\lambda_1}{2}$$

$$z_{\min} = \frac{(2n+1)\lambda_1}{4}$$

For $\Gamma < 0$:

$$z_{\max} = \frac{(2n+1)\lambda_1}{4}$$

$$z_{\min} = \frac{n\lambda_1}{2}$$

Hence, standing waves are produced in medium 1 due to interference of incident and reflected waves.

Q4. Derive Snell’s law and wave vector relations for oblique incidence.

Answer:

Consider an electromagnetic wave obliquely incident at the interface $z = 0$ between two lossless dielectric media.

The plane of incidence is the xz -plane.

For incident wave:

$$\mathbf{k}_i = \beta_1 \sin\theta_i \hat{a}_x + \beta_1 \cos\theta_i \hat{a}_z$$

For reflected wave:

$$\mathbf{k}_r = \beta_1 \sin\theta_r \hat{a}_x - \beta_1 \cos\theta_r \hat{a}_z$$

For transmitted wave:

$$\mathbf{k}_t = \beta_2 \sin\theta_t \hat{a}_x + \beta_2 \cos\theta_t \hat{a}_z$$

At the boundary, the phase of all waves must be same along the interface.

The tangential component of propagation constant must be continuous:

$$\beta_1 \sin\theta_i = \beta_1 \sin\theta_r$$

Therefore:

$$\boxed{\theta_i = \theta_r}$$

This is the law of reflection.

Also:

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

Since:

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$\omega\sqrt{\mu_1\varepsilon_1} \sin\theta_i = \omega\sqrt{\mu_2\varepsilon_2} \sin\theta_t$$

$$\boxed{\sqrt{\mu_1\varepsilon_1} \sin\theta_i = \sqrt{\mu_2\varepsilon_2} \sin\theta_t}$$

This is Snell’s law in electromagnetic form.

If refractive index is used:

$$n = \sqrt{\mu_r \varepsilon_r}$$

then:

$$\boxed{n_1 \sin \theta_i = n_2 \sin \theta_t}$$

The wave vector magnitudes are:

$$|\mathbf{k}_i| = \beta_1$$

$$|\mathbf{k}_r| = \beta_1$$

$$|\mathbf{k}_t| = \beta_2$$

Thus, for oblique incidence:

$$\boxed{\theta_i = \theta_r}$$

$$\boxed{\beta_1 \sin \theta_i = \beta_2 \sin \theta_t}$$

Q5. Derive Fresnel reflection and transmission coefficients for perpendicular polarization.

Answer:

In perpendicular polarization, the electric field is perpendicular to the plane of incidence.

Here, the plane of incidence is xz -plane, so:

$$\mathbf{E} \parallel \hat{a}_y$$

Incident electric field:

$$\mathbf{E}_i = E_{i0} e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)} \hat{a}_y$$

Reflected electric field:

$$\mathbf{E}_r = E_{r0} e^{-j\beta_1(x \sin\theta_i - z \cos\theta_i)} \hat{a}_y$$

Transmitted electric field:

$$\mathbf{E}_t = E_{t0} e^{-j\beta_2(x \sin\theta_t + z \cos\theta_t)} \hat{a}_y$$

The magnetic fields are:

$$\mathbf{H}_i = \frac{E_{i0}}{\eta_1}\left(-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right) e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)}$$

$$\mathbf{H}_r = \frac{E_{r0}}{\eta_1}\left(\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right) e^{-j\beta_1(x \sin\theta_i - z \cos\theta_i)}$$

$$\mathbf{H}_t = \frac{E_{t0}}{\eta_2}\left(-\cos\theta_t \hat{a}_x + \sin\theta_t \hat{a}_z\right) e^{-j\beta_2(x \sin\theta_t + z \cos\theta_t)}$$

At $z = 0$, tangential electric field is continuous:

$$E_{i0} + E_{r0} = E_{t0}$$

$$1 + \Gamma_\perp = \tau_\perp$$

Tangential magnetic field is also continuous:

$$H_{xi} + H_{xr} = H_{xt}$$

$$-\frac{E_{i0}}{\eta_1}\cos\theta_i + \frac{E_{r0}}{\eta_1}\cos\theta_i = -\frac{E_{t0}}{\eta_2}\cos\theta_t$$

Multiplying by -1 :

$$\frac{E_{i0}}{\eta_1}\cos\theta_i - \frac{E_{r0}}{\eta_1}\cos\theta_i = \frac{E_{t0}}{\eta_2}\cos\theta_t$$

$$\frac{E_{i0} - E_{r0}}{\eta_1}\cos\theta_i = \frac{E_{t0}}{\eta_2}\cos\theta_t$$

Divide by E_{i0} :

$$\frac{1-\Gamma_\perp}{\eta_1}\cos\theta_i = \frac{\tau_\perp}{\eta_2}\cos\theta_t$$

Using:

$$\tau_\perp = 1+\Gamma_\perp$$

$$\frac{1-\Gamma_\perp}{\eta_1}\cos\theta_i = \frac{1+\Gamma_\perp}{\eta_2}\cos\theta_t$$

$$\eta_2\cos\theta_i(1-\Gamma_\perp) = \eta_1\cos\theta_t(1+\Gamma_\perp)$$

$$\eta_2\cos\theta_i - \eta_2\cos\theta_i\Gamma_\perp = \eta_1\cos\theta_t + \eta_1\cos\theta_t\Gamma_\perp$$

$$\eta_2\cos\theta_i - \eta_1\cos\theta_t = \Gamma_\perp\left(\eta_2\cos\theta_i + \eta_1\cos\theta_t\right)$$

Therefore:

$$\boxed{\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

Transmission coefficient:

$$\tau_\perp = 1+\Gamma_\perp$$

$$\boxed{\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

These are Fresnel coefficients for perpendicular polarization.

Q6. Derive Fresnel reflection and transmission coefficients for parallel polarization.

Answer:

In parallel polarization, electric field lies in the plane of incidence.

The plane of incidence is xz -plane. Therefore electric field has x and z components, and magnetic field is along y -direction.

Incident electric field:

$$\mathbf{E}_i = E_{i0}\left(\cos\theta_i \hat{a}_x - \sin\theta_i \hat{a}_z\right) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

Reflected electric field:

$$\mathbf{E}_r = E_{r0}\left(\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right) e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

Transmitted electric field:

$$\mathbf{E}_t = E_{t0}\left(\cos\theta_t \hat{a}_x - \sin\theta_t \hat{a}_z\right) e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

The magnetic fields are:

$$\mathbf{H}_i = \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \hat{a}_y$$

$$\mathbf{H}_r = -\frac{E_{r0}}{\eta_1} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \hat{a}_y$$

$$\mathbf{H}_t = \frac{E_{t0}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a}_y$$

At $z = 0$, tangential electric field is continuous:

$$E_{i0}\cos\theta_i + E_{r0}\cos\theta_i = E_{t0}\cos\theta_t$$

$$(E_{i0} + E_{r0})\cos\theta_i = E_{t0}\cos\theta_t$$

Dividing by E_{i0} :

$$(1 + \Gamma_{\parallel})\cos\theta_i = \tau_{\parallel}\cos\theta_t$$

Tangential magnetic field is continuous:

$$H_{iy} + H_{ry} = H_{ty}$$

$$\frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$$

$$\frac{1 - \Gamma_{\|}}{\eta_1} = \frac{\tau_{\|}}{\eta_2}$$

Therefore:

$$\tau_{\|} = \frac{\eta_2}{\eta_1}(1 - \Gamma_{\|})$$

Substitute this in electric field boundary condition:

$$(1 + \Gamma_{\|}) \cos\theta_i = \frac{\eta_2}{\eta_1}(1 - \Gamma_{\|}) \cos\theta_t$$

$$\eta_1 \cos\theta_i(1 + \Gamma_{\|}) = \eta_2 \cos\theta_t(1 - \Gamma_{\|})$$

$$\eta_1 \cos\theta_i + \eta_1 \cos\theta_i \Gamma_{\|} = \eta_2 \cos\theta_t - \eta_2 \cos\theta_t \Gamma_{\|}$$

$$\Gamma_{\|} \left(\eta_1 \cos\theta_i + \eta_2 \cos\theta_t\right) = \eta_2 \cos\theta_t - \eta_1 \cos\theta_i$$

Therefore:

$$\boxed{\Gamma_{\|} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

Transmission coefficient:

$$\tau_{\|} = \frac{\eta_2}{\eta_1}(1 - \Gamma_{\|})$$

$$\boxed{\tau_{\|} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

These are Fresnel coefficients for parallel polarization.

Q7. Derive Brewster angle for perfect dielectrics.

Answer:

Brewster angle is defined as the angle of incidence at which reflection coefficient becomes zero.

$$\boxed{\Gamma = 0}$$

For parallel polarization, reflection coefficient is:

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

At Brewster angle:

$$\theta_i = \theta_B$$

and

$$\Gamma_{\parallel} = 0$$

Therefore numerator must be zero:

$$\eta_2 \cos\theta_t - \eta_1 \cos\theta_B = 0$$

$$\eta_2 \cos\theta_t = \eta_1 \cos\theta_B$$

$$\boxed{\frac{\cos\theta_t}{\cos\theta_B} = \frac{\eta_1}{\eta_2}}$$

Using:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\frac{\eta_1}{\eta_2} = \sqrt{\frac{\mu_1/\varepsilon_1}{\mu_2/\varepsilon_2}} = \sqrt{\frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}$$

Hence:

$$\cos\theta_t = \sqrt{\frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}\cos\theta_B$$

From Snell's law:

$$\sqrt{\mu_1\varepsilon_1}\sin\theta_B = \sqrt{\mu_2\varepsilon_2}\sin\theta_t$$

For non-magnetic media:

$$\mu_1 = \mu_2 = \mu_0$$

Then Snell's law becomes:

$$\sqrt{\varepsilon_1}\sin\theta_B = \sqrt{\varepsilon_2}\sin\theta_t$$

At Brewster angle, the reflected and transmitted rays are perpendicular:

$$\theta_B + \theta_t = 90^\circ$$

Therefore:

$$\theta_t = 90^\circ - \theta_B$$

$$\sin\theta_t = \cos\theta_B$$

Using Snell's law:

$$\sqrt{\varepsilon_1}\sin\theta_B = \sqrt{\varepsilon_2}\cos\theta_B$$

$$\frac{\sin\theta_B}{\cos\theta_B} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\boxed{\tan\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

Therefore:

$$\boxed{\theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

Equivalent form:

$$\boxed{\theta_B = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}}$$

Thus, for non-magnetic perfect dielectrics:

$$\boxed{\theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

Q8. Derive critical angle and condition for total internal reflection.

Answer:

Critical angle is the angle of incidence in the denser medium for which the angle of refraction becomes 90° .

From Snell's law:

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

At critical angle:

$$\theta_i = \theta_c$$

$$\theta_t = 90^\circ$$

Therefore:

$$\sin\theta_t = \sin 90^\circ = 1$$

So:

$$\beta_1 \sin\theta_c = \beta_2$$

$$\boxed{\sin\theta_c = \frac{\beta_2}{\beta_1}}$$

Since:

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$\sin\theta_c = \frac{\omega\sqrt{\mu_2\varepsilon_2}}{\omega\sqrt{\mu_1\varepsilon_1}}$$

$$\boxed{\sin\theta_c = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}}$$

For non-magnetic media:

$$\mu_1 = \mu_2 = \mu_0$$

Therefore:

$$\boxed{\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

$$\boxed{\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

This is possible only when:

$$\varepsilon_1 > \varepsilon_2$$

or

$$\beta_1 > \beta_2$$

That means the wave must travel from denser medium to rarer medium.

For total internal reflection:

$$\boxed{\theta_i > \theta_c}$$

In this case, the transmitted angle becomes imaginary and no real power is transmitted into the second medium. The wave in medium 2 becomes an evanescent wave.

Hence:

$$\boxed{\text{Total internal reflection occurs when } \theta_i > \theta_c}$$

Q9. Write field equations for oblique incidence in perpendicular polarization.

Answer:

For perpendicular polarization:

$$\mathbf{E} \perp \text{plane of incidence}$$

The plane of incidence is xz -plane, therefore:

$$\mathbf{E} \parallel \hat{a}_y$$

For incident wave:

$$\mathbf{k}_i = \beta_1 \sin\theta_i \hat{a}_x + \beta_1 \cos\theta_i \hat{a}_z$$

Therefore:

$$\mathbf{k}_i \cdot \mathbf{r} = \beta_1 x \sin\theta_i + \beta_1 z \cos\theta_i$$

Incident electric field:

$$\boxed{\mathbf{E}_i = E_{i0} e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)} \hat{a}_y}$$

Incident magnetic field:

$$\boxed{\mathbf{H}_i = \frac{E_{i0}}{\eta_1} \left(-\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right) e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)}}$$

For reflected wave:

$$\mathbf{k}_r = \beta_1 \sin\theta_i \hat{a}_x - \beta_1 \cos\theta_i \hat{a}_z$$

$$\mathbf{k}_r \cdot \mathbf{r} = \beta_1 x \sin\theta_i - \beta_1 z \cos\theta_i$$

Reflected electric field:

$$\boxed{\mathbf{E}_r = E_{r0} e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)} \hat{a}_y}$$

Reflected magnetic field:

$$\boxed{\mathbf{H}_r = \frac{E_{r0}}{\eta_1} \left(\cos\theta_i \hat{a}_x + \sin\theta_i \hat{a}_z\right) e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)}}$$

For transmitted wave:

$$\mathbf{k}_t = \beta_2 \sin\theta_t \hat{a}_x + \beta_2 \cos\theta_t \hat{a}_z$$

$$\mathbf{k}_t \cdot \mathbf{r} = \beta_2 x \sin\theta_t + \beta_2 z \cos\theta_t$$

Transmitted electric field:

$$\boxed{\mathbf{E}_t = E_{t0} e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)} \hat{a}_y}$$

Transmitted magnetic field:

$$\boxed{\mathbf{H}_t = \frac{E_{t0}}{\eta_2} \left(-\cos\theta_t \hat{a}_x + \sin\theta_t \hat{a}_z\right) e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)}}$$

These are the complete field equations for perpendicular polarization.

Q10. Write field equations for oblique incidence in parallel polarization.

Answer:

For parallel polarization, electric field lies in the plane of incidence.

The plane of incidence is xz -plane. Therefore, electric field has x and z components, while magnetic field is along y -direction.

Incident electric field:

$$\boxed{\mathbf{E}_i = E_{i0}\left(\cos\theta_i\hat{a}_x - \sin\theta_i\hat{a}_z\right)e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}}$$

Incident magnetic field:

$$\boxed{\mathbf{H}_i = \frac{E_{i0}}{\eta_1}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\hat{a}_y}$$

Reflected electric field:

$$\boxed{\mathbf{E}_r = E_{r0}\left(\cos\theta_i\hat{a}_x + \sin\theta_i\hat{a}_z\right)e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}}$$

Reflected magnetic field:

$$\boxed{\mathbf{H}_r = -\frac{E_{r0}}{\eta_1}e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}\hat{a}_y}$$

Transmitted electric field:

$$\boxed{\mathbf{E}_t = E_{t0}\left(\cos\theta_t\hat{a}_x - \sin\theta_t\hat{a}_z\right)e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}}$$

Transmitted magnetic field:

$$\boxed{\mathbf{H}_t = \frac{E_{t0}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a}_y}$$

The Fresnel coefficients for this case are:

$$\boxed{\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}}$$

$$\boxed{\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}}$$

Q11. Explain surface impedance and derive its expression for a conductor.

Answer:

At high frequencies, current in a conductor is concentrated near the surface due to skin effect.

The current density decreases exponentially inside the conductor.

Let the current density be:

$$J = J_0 e^{-\gamma y}$$

where:

$$\gamma = \alpha + j\beta$$

The linear surface current density is:

$$J_s = \int_0^{\infty} J dy$$

$$J_s = \int_0^\infty J_0 e^{-\gamma y} dy$$

$$J_s = J_0 \left[\frac{e^{-\gamma y}}{-\gamma} \right]_0^\infty$$

$$J_s = \frac{J_0}{\gamma}$$

But conduction current density is:

$$J_0 = \sigma E_{\tan}$$

Therefore:

$$J_s = \frac{\sigma E_{\tan}}{\gamma}$$

Surface impedance is defined as the ratio of tangential electric field at the conductor surface to the surface current density:

$$\boxed{Z_s = \frac{E_{\tan}}{J_s}}$$

Substitute J_s :

$$Z_s = \frac{E_{\tan}}{\sigma E_{\tan}/\gamma}$$

$$\boxed{Z_s = \frac{\gamma}{\sigma}}$$

For a conductor:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Therefore:

$$\boxed{Z_s = \frac{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}{\sigma}}$$

For a good conductor:

$$\sigma \gg \omega\varepsilon$$

So:

$$\gamma \approx \sqrt{j\omega\mu\sigma}$$

$$\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$$

Therefore:

$$Z_s = \frac{(1+j)\sqrt{\frac{\omega\mu\sigma}{2}}}{\sigma}$$

$$\boxed{Z_s = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}}$$

So:

$$\boxed{Z_s = R_s + jX_s}$$

where:

$$\boxed{R_s = X_s = \sqrt{\frac{\omega\mu}{2\sigma}}}$$

Here R_s is surface resistance and X_s is surface reactance.

Q12. Derive Poynting theorem.

Answer:

Poynting theorem states that net power flowing out of a given volume is equal to the rate of decrease of electromagnetic energy stored in the volume minus the ohmic losses.

From Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

For a linear medium:

$$\mathbf{B} = \mu \mathbf{H}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{J} = \sigma \mathbf{E}$$

So:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

Dot multiply first equation by $\mathbf{H}$:

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = -\mu \mathbf{H} \cdot \frac{\partial \mathbf{H}}{\partial t}$$

Dot multiply second equation by $\mathbf{E}$:

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = \sigma E^2 + \varepsilon \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t}$$

Using vector identity:

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H})$$

Substitute:

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\mu \mathbf{H} \cdot \frac{\partial \mathbf{H}}{\partial t} - \sigma E^2 - \varepsilon \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t}$$

Now:

$$\mathbf{H} \cdot \frac{\partial \mathbf{H}}{\partial t} = \frac{1}{2} \frac{\partial H^2}{\partial t}$$

$$\mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t}$$

Therefore:

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \frac{\varepsilon}{2} \frac{\partial E^2}{\partial t} - \sigma E^2$$

$$\boxed{\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) - \sigma E^2}$$

Integrating over volume V:

$$\int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}) dv = -\frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) dv - \int_V \sigma E^2 dv$$

Using divergence theorem:

$$\int_V \nabla \cdot (\mathbf{E} \times \mathbf{H}) dv = \oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S}$$

Hence:

$$\boxed{\oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2 \right) dv - \int_V \sigma E^2 dv}$$

This is Poynting theorem.

Where:

$$\boxed{\mathbf{P} = \mathbf{E} \times \mathbf{H}}$$

is Poynting vector.

Q13. Derive time-average Poynting vector for a uniform plane wave.

Answer:

Poynting vector is defined as:

$$\boxed{\mathbf{P} = \mathbf{E} \times \mathbf{H}}$$

Consider a wave travelling in $+z$ -direction in a lossy medium:

$$\mathbf{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_x$$

The magnetic field is:

$$\mathbf{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \hat{a}_y$$

where:

$$H_0 = \frac{E_0}{|\eta|}$$

Thus:

$$\mathbf{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \hat{a}_y$$

Poynting vector:

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

$$\mathbf{P} = \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta) \hat{a}_z$$

Using:

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\mathbf{P} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} [\cos \theta_\eta + \cos(2\omega t - 2\beta z - \theta_\eta)] \hat{a}_z$$

The second term is AC term whose average over one cycle is zero.

Therefore, time-average Poynting vector is:

$$\boxed{\mathbf{P}_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos \theta_\eta \, \hat{a}_z}$$

For lossless medium:

$$\alpha = 0$$

$$\eta \text{ is real}$$

$$\theta_\eta = 0$$

So:

$$\boxed{\mathbf{P}_{avg} = \frac{E_0^2}{2\eta} \hat{a}_z}$$

The total power crossing a surface S is:

$$\boxed{P_{avg} = \int_S \mathbf{P}_{avg} \cdot d\mathbf{S}}$$

Q14. Derive power loss in a plane conductor using surface impedance.

Answer:

For a good conductor, current is concentrated near the surface due to skin effect.

The surface impedance is:

$$Z_s = R_s + jX_s$$

For a good conductor:

$$\boxed{R_s = X_s = \sqrt{\frac{\omega\mu}{2\sigma}}}$$

The tangential electric field at the conductor surface is related to surface current density by:

$$E_{\tan} = Z_s J_s$$

At the conductor surface:

$$J_s = \hat{n} \times H_{\tan}$$

Thus, magnitude:

$$|J_s| = |H_{\tan}|$$

The average power entering the conductor per unit area is:

$$P_L = \frac{1}{2}\Re(E_{\tan} J_s^*)$$

Substitute:

$$E_{\tan} = Z_s J_s$$

$$P_L = \frac{1}{2}\Re(Z_s J_s J_s^*)$$

$$P_L = \frac{1}{2}\Re(Z_s)|J_s|^2$$

Since:

$$\Re(Z_s) = R_s$$

$$P_L = \frac{1}{2}R_s|J_s|^2$$

But:

$$|J_s| = |H_{\tan}|$$

Therefore:

$$\boxed{P_L = \frac{1}{2}R_s|H_{\tan}|^2}$$

For a surface area S:

$$\boxed{P_L = \frac{1}{2}R_s\int_S |H_{\tan}|^2 dS}$$

This is the power loss in a plane conductor.

Q15. Explain plane wave reflection at oblique incidence from a perfect conductor.

Answer:

For a perfect conductor:

$$\sigma \to \infty$$

Therefore the field inside the conductor is zero:

$$\boxed{\mathbf{E}_t = 0}$$

At the surface of perfect conductor:

$$\boxed{\mathbf{E}_{\tan} = 0}$$

This means the tangential component of total electric field at the boundary must be zero.

For oblique incidence, the total tangential electric field is:

$$\mathbf{E}_{\tan,total} = \mathbf{E}_{\tan,i} + \mathbf{E}_{\tan,r}$$

At conductor surface:

$$\mathbf{E}_{\tan,i} + \mathbf{E}_{\tan,r} = 0$$

Therefore:

$$\mathbf{E}_{\tan,r} = -\mathbf{E}_{\tan,i}$$

Hence reflection coefficient for tangential electric field is:

$$\boxed{\Gamma = -1}$$

Also, transmitted wave is zero:

$$\boxed{\tau = 0}$$

For normal incidence also:

$$\Gamma = -1$$

$$\tau = 0$$

Thus, a perfect conductor reflects the entire incident wave.

The incident and reflected waves form standing waves in the dielectric region.

For normal incidence, standing wave electric field is:

$$\boxed{\mathbf{E}_s = 2E_{i0}\sin\beta z\sin\omega t\ \hat{a}_x}$$

Magnetic field is:

$$\boxed{\mathbf{H}_s = 2H_{i0}\cos\beta z\cos\omega t\ \hat{a}_y}$$

At the conductor surface $z = 0$:

$$\mathbf{E}_s = 0$$

$$\mathbf{H}_s = 2H_{i0}\cos\omega t\hat{a}_y$$

Therefore, electric field has a node and magnetic field has an antinode at the conductor surface.

IMPORTANT FORMULA LIST FOR EXAM

Normal Incidence

$$\boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\boxed{\tau = 1 + \Gamma = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

Perfect Conductor

$$\boxed{\eta_2 = 0}$$

$$\boxed{\Gamma = -1}$$

$$\boxed{\tau = 0}$$

Perpendicular Polarization

$$\boxed{\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$$\boxed{\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

Parallel Polarization

$$\boxed{\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

$$\boxed{\tau_\parallel = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

Snell's Law

$$\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

Brewster Angle

$$\boxed{\theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

For non-magnetic media.

Critical Angle

$$\boxed{\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$

For non-magnetic media, when $\varepsilon_1 > \varepsilon_2$.

Surface Impedance

$$\boxed{Z_s = \frac{E_{\tan}}{J_s}}$$

$$\boxed{Z_s = \frac{\gamma}{\sigma}}$$

For good conductor:

$$\boxed{Z_s = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}}$$

Poynting Vector

$$\boxed{\mathbf{P} = \mathbf{E} \times \mathbf{H}}$$

Average Poynting Vector

$$\boxed{\mathbf{P}_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta \hat{a}_z}$$

Power Loss in Plane Conductor

$$\boxed{P_L = \frac{1}{2} R_s |H_{\tan}|^2}$$

NUMERICAL PROBLEMS AT THE END

Problem 1. A wave in free space is given by

$$\mathbf{E}_i = 100 e^{-j(0.866y + 0.5z)} \hat{a}_x \ \mathrm{V/m}$$

Find:

1. ω
2. λ
3. $\mathbf{H}_i$

Solution:

Given:

$$\mathbf{E}_i = 100 e^{-j(0.866y + 0.5z)} \hat{a}_x$$

The propagation vector is:

$$\mathbf{k} = 0.866 \hat{a}_y + 0.5 \hat{a}_z$$

Magnitude:

$$\beta = |\mathbf{k}|$$

$$\beta = \sqrt{(0.866)^2 + (0.5)^2}$$

$$\beta = \sqrt{0.749 + 0.25}$$

$$\beta \approx 1 \text{ rad/m}$$

For free space:

$$\beta = \frac{\omega}{c}$$

$$\omega = \beta c$$

$$\omega = 1 \times 3 \times 10^8$$

$$\boxed{\omega = 3 \times 10^8 \text{ rad/s}}$$

Wavelength:

$$\lambda = \frac{2\pi}{\beta}$$

$$\lambda = \frac{2\pi}{1}$$

$$\boxed{\lambda = 2\pi \text{ m}}$$

Unit propagation vector:

$$\hat{a}_k = 0.866\hat{a}_y + 0.5\hat{a}_z$$

For plane wave:

$$\mathbf{H} = \frac{1}{\eta_0}\hat{a}_k \times \mathbf{E}$$

$$\eta_0 = 120\pi \,\Omega$$

$$\mathbf{H}_i = \frac{100}{120\pi}\left[(0.866\hat{a}_y + 0.5\hat{a}_z) \times \hat{a}_x\right] e^{-j(0.866y+0.5z)}$$

Now:

$$\hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$\hat{a}_z \times \hat{a}_x = \hat{a}_y$$

Therefore:

$$\mathbf{H}_i = \frac{100}{120\pi}\left[-0.866\hat{a}_z + 0.5\hat{a}_y\right]e^{-j(0.866y+0.5z)}$$

$$\boxed{\mathbf{H}_i = \frac{5}{6\pi}\left[0.5\hat{a}_y - 0.866\hat{a}_z\right]e^{-j(0.866y+0.5z)}\ \mathrm{A/m}}$$

Problem 2: Normal Incidence on a Lossless Dielectric

A uniform plane wave of frequency

$$f = 5\ \mathrm{GHz}$$

is normally incident from free space onto a lossless dielectric medium having

$$\varepsilon_2 = 4\varepsilon_0, \qquad \mu_2 = \mu_0$$

Assume the incident electric field is

$$\vec{E}_i = \hat{x}E_0 e^{-j\beta_1 z}\ \mathrm{V/m}$$

Find the reflected and transmitted fields.

Solution

Step 1: Intrinsic impedances of the two media

For free space,

$$\eta_1 = \eta_0 = 120\pi\ \Omega$$

For medium 2,

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Given,

$$\mu_2 = \mu_0, \qquad \varepsilon_2 = 4\varepsilon_0$$

Therefore,

$$\eta_2 = \sqrt{\frac{\mu_0}{4\varepsilon_0}}$$

$$\eta_2 = \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\eta_2 = \frac{\eta_0}{2}$$

$$\eta_2 = 60\pi\ \Omega$$

So,

$$\boxed{\eta_1 = 120\pi\ \Omega}$$

$$\boxed{\eta_2 = 60\pi\ \Omega}$$

Step 2: Reflection coefficient

For normal incidence, the reflection coefficient is

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Substituting values,

$$\Gamma = \frac{60\pi - 120\pi}{60\pi + 120\pi}$$

$$\Gamma = \frac{-60\pi}{180\pi}$$

$$\boxed{\Gamma = -\frac{1}{3}}$$

Therefore, the reflected electric field amplitude is

$$E_r = \Gamma E_0$$

$$E_r = -\frac{E_0}{3}$$

Step 3: Transmission coefficient

The transmission coefficient for electric field is

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Substituting,

$$\tau = \frac{2(60\pi)}{120\pi + 60\pi}$$

$$\tau = \frac{120\pi}{180\pi}$$

$$\boxed{\tau = \frac{2}{3}}$$

Therefore,

$$E_t = \tau E_0$$

$$E_t = \frac{2E_0}{3}$$

Step 4: Phase constants

The phase constant is

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Also,

$$\beta = \frac{2\pi}{\lambda}$$

For free space,

$$\beta_1 = \frac{\omega}{c}$$

Since,

$$f = 5 \times 10^9 \text{ Hz}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi(5 \times 10^9)$$

$$\omega = 10\pi \times 10^9 \text{ rad/s}$$

Now,

$$\beta_1 = \frac{2\pi f}{c}$$

$$\beta_1 = \frac{2\pi(5 \times 10^9)}{3 \times 10^8}$$

$$\beta_1 = \frac{10\pi}{0.3}$$

$$\beta_1 = 104.72 \text{ rad/m}$$

So,

$$\boxed{\beta_1 = 104.72 \text{ rad/m}}$$

For medium 2,

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\beta_2 = \omega\sqrt{\mu_0(4\varepsilon_0)}$$

$$\beta_2 = 2\omega\sqrt{\mu_0\varepsilon_0}$$

$$\beta_2 = 2\beta_1$$

$$\beta_2 = 2(104.72)$$

$$\boxed{\beta_2 = 209.44 \text{ rad/m}}$$

Step 5: Incident field

Given,

$$\vec{E}_i = \hat{x}E_0 e^{-j\beta_1 z}$$

Substituting β_1 ,

$$\boxed{\vec{E}_i = \hat{x}E_0 e^{-j104.72z} \text{ V/m}}$$

For a wave travelling in $+z$ direction,

$$\vec{H}_i = \frac{1}{\eta_1}\hat{z} \times \vec{E}_i$$

Since,

$$\hat{z} \times \hat{x} = \hat{y}$$

Therefore,

$$\vec{H}_i = \hat{y}\frac{E_0}{\eta_1}e^{-j\beta_1 z}$$

$$\boxed{\vec{H}_i = \hat{y}\frac{E_0}{120\pi}e^{-j104.72z}\ \mathrm{A/m}}$$

Step 6: Reflected field

The reflected electric field is

$$\vec{E}_r = \hat{x}\Gamma E_0 e^{+j\beta_1 z}$$

Here the reflected wave travels in the $-z$ direction, so its phase factor is

$$e^{+j\beta_1 z}$$

Using,

$$\Gamma = -\frac{1}{3}$$

we get

$$\boxed{\vec{E}_r = -\hat{x}\frac{E_0}{3}e^{+j104.72z}\ \mathrm{V/m}}$$

For the reflected magnetic field,

$$\vec{H}_r = -\hat{y}\frac{\Gamma E_0}{\eta_1}e^{+j\beta_1 z}$$

Substitute $\Gamma = -1/3$,

$$\vec{H}_r = -\hat{y}\frac{\left(-\frac{1}{3}\right)E_0}{120\pi}e^{+j104.72z}$$

$$\vec{H}_r = \hat{y}\frac{E_0}{360\pi}e^{+j104.72z}$$

Therefore,

$$\boxed{\vec{H}_r = \hat{y}\frac{E_0}{360\pi}e^{+j104.72z}\ \mathrm{A/m}}$$

Step 7: Transmitted field

The transmitted electric field is

$$\vec{E}_t = \hat{x}\tau E_0 e^{-j\beta_2 z}$$

Using,

$$\tau = \frac{2}{3}$$

and

$$\beta_2 = 209.44\ \mathrm{rad/m}$$

we get

$$\boxed{\vec{E}_t = \hat{x}\frac{2E_0}{3}e^{-j209.44z}\ \mathrm{V/m}}$$

For the transmitted magnetic field,

$$\vec{H}_t = \hat{y}\frac{E_t}{\eta_2}$$

$$\vec{H}_t = \hat{y}\frac{\frac{2E_0}{3}}{60\pi}e^{-j209.44z}$$

$$\vec{H}_t = \hat{y}\frac{2E_0}{180\pi}e^{-j209.44z}$$

$$\boxed{\vec{H}_t = \hat{y}\frac{E_0}{90\pi}e^{-j209.44z}\ \mathrm{A/m}}$$

Final Answer

$$\boxed{\Gamma = -\frac{1}{3}}$$

$$\boxed{\tau = \frac{2}{3}}$$

$$\boxed{\beta_1 = 104.72 \text{ rad/m}}$$

$$\boxed{\beta_2 = 209.44 \text{ rad/m}}$$

Incident field:

$$\boxed{\vec{E}_i = \hat{x} E_0 e^{-j104.72z} \text{ V/m}}$$

$$\boxed{\vec{H}_i = \hat{y} \frac{E_0}{120\pi} e^{-j104.72z} \text{ A/m}}$$

Reflected field:

$$\boxed{\vec{E}_r = -\hat{x} \frac{E_0}{3} e^{+j104.72z} \text{ V/m}}$$

$$\boxed{\vec{H}_r = \hat{y} \frac{E_0}{360\pi} e^{+j104.72z} \text{ A/m}}$$

Transmitted field:

$$\boxed{\vec{E}_t = \hat{x} \frac{2E_0}{3} e^{-j209.44z} \text{ V/m}}$$

$$\boxed{\vec{H}_t = \hat{y} \frac{E_0}{90\pi} e^{-j209.44z} \text{ A/m}}$$

Power Check

The reflected power coefficient is

$$|\Gamma|^2 = \left(-\frac{1}{3}\right)^2$$

$$|\Gamma|^2 = \frac{1}{9}$$

So,

$$\boxed{\text{Reflected power} = 11.11\%}$$

The transmitted power coefficient is

$$T = 1 - |\Gamma|^2$$

$$T = 1 - \frac{1}{9}$$

$$T = \frac{8}{9}$$

$$\boxed{\text{Transmitted power} = 88.89\%}$$

Thus, most of the incident power enters the dielectric medium.

If $E_0 = 100 \text{ V/m}$, then

Incident electric field:

$$\vec{E}_i = \hat{x}100e^{-j104.72z} \text{ V/m}$$

Reflected electric field:

$$\vec{E}_r = -\hat{x}\frac{100}{3}e^{+j104.72z}$$

$$\boxed{\vec{E}_r = -\hat{x}33.33e^{+j104.72z} \text{ V/m}}$$

Transmitted electric field:

$$\vec{E}_t = \hat{x}\frac{2(100)}{3}e^{-j209.44z}$$

$$\boxed{\vec{E}_t = \hat{x}66.67e^{-j209.44z} \text{ V/m}}$$

Incident magnetic field:

$$\vec{H}_i = \hat{y}\frac{100}{120\pi}e^{-j104.72z}$$

$$\boxed{\vec{H}_i = \hat{y}0.265e^{-j104.72z} \text{ A/m}}$$

Reflected magnetic field:

$$\vec{H}_r = \hat{y}\frac{100}{360\pi}e^{+j104.72z}$$

$$\boxed{\vec{H}_r = \hat{y}0.0884e^{+j104.72z} \text{ A/m}}$$

Transmitted magnetic field:

$$\vec{H}_t = \hat{y}\frac{100}{90\pi}e^{-j209.44z}$$

$$\boxed{\vec{H}_t = \hat{y}0.3537e^{-j209.44z} \text{ A/m}}$$

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