Below are **10-mark EMTL Unit-4 questions with full descriptive answers and derivations** based on your uploaded notes and syllabus. I have kept the **numericals/sums at the end only**.

UNIT–IV: TRANSMISSION LINES

10-Mark Questions and Answers

Q1. Explain transmission line parameters and derive the transmission line equations.

Answer:

At low frequencies, circuit elements are treated as lumped because voltage and current changes affect the entire circuit almost at the same time. But at high frequencies, the circuit elements are distributed throughout the length of the line. Hence a transmission line is represented using distributed parameters per unit length.

The primary constants of a transmission line are:

$$R = \text{Resistance per unit length } (\Omega/m)$$

$$L = \text{Inductance per unit length } (H/m)$$

$$G = \text{Conductance per unit length } (S/m)$$

$$C = \text{Capacitance per unit length } (F/m)$$

For a small section of line of length Δz, the equivalent circuit contains:

$$R\Delta z, \quad L\Delta z, \quad G\Delta z, \quad C\Delta z$$

Let the voltage and current at position z be:

$$V(z,t), \qquad I(z,t)$$

and at position $z + \Delta z$ be:

$$V(z+\Delta z,t), \qquad I(z+\Delta z,t)$$

Applying KVL

For the series branch:

$$V(z,t) - V(z+\Delta z,t) = R\Delta z I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t}$$

Rearranging:

$$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Taking limit as:

$$\Delta z \to 0$$

we get:

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Therefore,

$$\boxed{-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}}$$

This is the first transmission line equation.

Applying KCL

At the shunt branch:

$$I(z,t) - I(z+\Delta z,t) = G\Delta z V(z+\Delta z,t) + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

Rearranging:

$$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

Taking limit as:

$$\Delta z \to 0$$

we get:

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

Therefore,

$$\boxed{-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}}$$

This is the second transmission line equation.

Hence the time-domain transmission line equations are:

$$\boxed{-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}}$$

$$\boxed{-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}}$$

For sinusoidal steady state, replacing:

$$\frac{\partial}{\partial t} = j\omega$$

we get:

$$\boxed{-\frac{dV}{dz} = (R + j\omega L)I}$$

$$\boxed{-\frac{dI}{dz} = (G + j\omega C)V}$$

These are the phasor form transmission line equations.

Q2. Derive the wave equations for voltage and current on a transmission line.

Answer:

The phasor transmission line equations are:

$$-\frac{dV}{dz} = (R + j\omega L)I$$

$$-\frac{dI}{dz} = (G + j\omega C)V$$

Differentiate the first equation with respect to z:

$$-\frac{d^2V}{dz^2} = (R + j\omega L)\frac{dI}{dz}$$

From the second equation:

$$\frac{dI}{dz} = -(G + j\omega C)V$$

Substitute this in the above equation:

$$-\frac{d^2V}{dz^2} = (R + j\omega L)\left[-(G + j\omega C)V\right]$$

$$\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V$$

Therefore,

$$\frac{d^2V}{dz^2} - \gamma^2 V = 0$$

where

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

is the propagation constant.

Hence,

$$\boxed{\frac{d^2V}{dz^2} - \gamma^2 V = 0}$$

Similarly, differentiating the second transmission line equation:

$$-\frac{dI}{dz} = (G + j\omega C)V$$

with respect to z:

$$-\frac{d^2I}{dz^2} = (G + j\omega C)\frac{dV}{dz}$$

From the first equation:

$$\frac{dV}{dz} = -(R + j\omega L)I$$

Substitute:

$$-\frac{d^2I}{dz^2} = (G + j\omega C)\left[-(R + j\omega L)I\right]$$

$$\frac{d^2I}{dz^2} = (R + j\omega L)(G + j\omega C)I$$

Therefore,

$$\boxed{\frac{d^2 I}{dz^2} - \gamma^2 I = 0}$$

Thus, the voltage and current wave equations are:

$$\boxed{\frac{d^2 V}{dz^2} - \gamma^2 V = 0}$$

$$\boxed{\frac{d^2 I}{dz^2} - \gamma^2 I = 0}$$

The general solutions are:

$$\boxed{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}}$$

$$\boxed{I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}}$$

where $V_0^+ e^{-\gamma z}$ represents the incident wave travelling in the $+z$ direction, and $V_0^- e^{+\gamma z}$ represents the reflected wave travelling in the $-z$ direction.

Q3. Define propagation constant and derive expressions for attenuation constant and phase constant.

Answer:

The propagation constant of a transmission line is defined as:

$$\boxed{\gamma = \alpha + j\beta}$$

where,

$$\alpha = \text{attenuation constant}$$

$$\beta = \text{phase constant}$$

The propagation constant is given by:

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

Squaring both sides:

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

Substitute:

$$\gamma = \alpha + j\beta$$

$$(\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$

Expanding left side:

$$(\alpha + j\beta)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$$

Expanding right side:

$$(R + j\omega L)(G + j\omega C)$$

$$= RG + j\omega RC + j\omega LG + j^2\omega^2 LC$$

$$= RG - \omega^2 LC + j\omega(RC + GL)$$

Comparing real and imaginary parts:

$$\boxed{\alpha^2 - \beta^2 = RG - \omega^2 LC}$$

$$\boxed{2\alpha\beta = \omega(RC + GL)}$$

Also,

$$|\gamma^2| = \sqrt{(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2}$$

But,

$$|\gamma^2| = |(\alpha + j\beta)^2| = \alpha^2 + \beta^2$$

Therefore,

$$\alpha^2 + \beta^2 = \sqrt{(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2}$$

Adding the equations:

$$(\alpha^2 + \beta^2) + (\alpha^2 - \beta^2) = \sqrt{(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2} + (RG - \omega^2 LC)$$

$$2\alpha^2 = \sqrt{(RG - \omega^2 LC)^2 + \omega^2(RC + GL)^2} + (RG - \omega^2 LC)$$

Thus,

$$\boxed{\alpha = \sqrt{\frac{1}{2}\left[RG - \omega^2 LC + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}\right]}}$$

Similarly, subtracting:

$$(\alpha^2 + \beta^2) - (\alpha^2 - \beta^2) = \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + GL)^2} - (RG - \omega^2 LC)$$

$$2\beta^2 = \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + GL)^2} - (RG - \omega^2 LC)$$

Therefore,

$$\boxed{\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)} - (RG - \omega^2 LC)\right]}}$$

Hence the propagation constant is:

$$\boxed{\gamma = \alpha + j\beta}$$

with:

$$\boxed{\alpha = \sqrt{\frac{1}{2}\left[RG - \omega^2 LC + \sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)}\right]}}$$

$$\boxed{\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2 L^2)(G^2+\omega^2 C^2)} - (RG - \omega^2 LC)\right]}}$$

Q4. Derive the characteristic impedance of a transmission line.

Answer:

The characteristic impedance of a transmission line is defined as the ratio of voltage to current of a travelling wave.

For a forward travelling wave:

$$V(z) = V_0^+ e^{-\gamma z}$$

$$I(z) = I_0^+ e^{-\gamma z}$$

The phasor transmission line equation is:

$$-\frac{dV}{dz} = (R + j\omega L)I$$

Differentiate:

$$V = V_0^+ e^{-\gamma z}$$

$$\frac{dV}{dz} = -\gamma V_0^+ e^{-\gamma z}$$

Therefore,

$$-\frac{dV}{dz} = \gamma V_0^+ e^{-\gamma z}$$

Using the transmission line equation:

$$\gamma V_0^+ e^{-\gamma z} = (R + j\omega L) I_0^+ e^{-\gamma z}$$

Cancel:

$$e^{-\gamma z}$$

$$\gamma V_0^+ = (R + j\omega L) I_0^+$$

Hence,

$$\frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma}$$

The characteristic impedance is:

$$Z_0 = \frac{V_0^+}{I_0^+}$$

Thus,

$$Z_0 = \frac{R + j\omega L}{\gamma}$$

But,

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Therefore,

$$Z_0 = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)^2}{(R + j\omega L)(G + j\omega C)}}$$

Hence,

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

This is the characteristic impedance of a general transmission line.

For a lossless line:

$$R = 0, \qquad G = 0$$

Therefore,

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Thus for a lossless line, the characteristic impedance is purely real.

Q5. Explain primary and secondary constants of transmission lines.

Answer:

Transmission line constants are divided into two categories:

1. Primary constants
2. Secondary constants

Primary Constants

Primary constants are the basic distributed parameters of a transmission line. They are measured per unit length.

They are:

$$\boxed{R}$$

$$R = \text{Resistance per unit length } (\Omega/m)$$

It represents the combined resistance of both conductors per unit length.

$$\boxed{L}$$

$$L = \text{Inductance per unit length } (H/m)$$

It represents the combined inductance of both conductors per unit length.

$$\boxed{C}$$

$$C = \text{Capacitance per unit length } (F/m)$$

It represents the capacitance between the two conductors per unit length.

$$\boxed{G}$$

$$G = \text{Conductance per unit length } (S/m)$$

It represents the conductance of the insulation medium between the conductors.

Secondary Constants

The constants derived from the primary constants are called secondary constants.

They are:

$$\boxed{Z_0}$$

Characteristic impedance:

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

$$\boxed{\gamma}$$

Propagation constant:

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

where,

$$\boxed{\gamma = \alpha + j\beta}$$

Here,

$$\alpha = \text{attenuation constant}$$

$$\beta = \text{phase constant}$$

For a Lossless Transmission Line

For a lossless line:

$$R = 0, \qquad G = 0$$

Propagation constant:

$$\gamma = \sqrt{(j\omega L)(j\omega C)}$$

$$\gamma = \sqrt{-\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

Hence,

$$\boxed{\alpha = 0}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

Characteristic impedance:

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Velocity of propagation:

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

Thus, the primary constants are R, L, G, C , and secondary constants are $Z_0, \gamma, \alpha, \beta$, and velocity.

Q6. Discuss lossless transmission line and derive its important expressions.

Answer:

A transmission line is said to be lossless if there is no power loss in the conductors and dielectric. Therefore:

$$\boxed{R = 0, \qquad G = 0}$$

The general propagation constant is:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

For a lossless line:

$$\gamma = \sqrt{(j\omega L)(j\omega C)}$$

$$\gamma = \sqrt{-\omega^2 LC}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

Since:

$$\gamma = \alpha + j\beta$$

Comparing:

$$\boxed{\alpha = 0}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

Hence, for a lossless line, attenuation is zero.

Characteristic Impedance

The general expression is:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

For a lossless line:

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Thus, Z_0 is purely real.

Phase Velocity

The phase velocity is:

$$v_p = \frac{\omega}{\beta}$$

Substituting:

$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

Voltage and Current Waves

For a forward travelling wave:

$$V(z,t) = V_0 e^{-\alpha z}\cos(\omega t - \beta z)$$

Since for lossless line:

$$\alpha = 0$$

Therefore,

$$\boxed{V(z,t) = V_0 \cos(\omega t - \beta z)}$$

Similarly,

$$\boxed{I(z,t) = I_0 \cos(\omega t - \beta z)}$$

Summary for Lossless Line

$$\boxed{R = 0, \qquad G = 0}$$

$$\boxed{\alpha = 0}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{\gamma = j\omega\sqrt{LC}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

Q7. State and derive the condition for a distortionless transmission line.

Answer:

A transmission line is said to be distortionless if all frequency components of a signal travel with the same velocity and suffer the same attenuation. For distortionless transmission, attenuation constant must be independent of frequency and phase constant must be directly proportional to frequency.

The general propagation constant is:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

For distortionless line, the condition is:

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

or,

$$\boxed{RC = LG}$$

Now,

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Factor R and G :

$$\gamma = \sqrt{R\left(1 + j\omega\frac{L}{R}\right)G\left(1 + j\omega\frac{C}{G}\right)}$$

$$\gamma = \sqrt{RG}\sqrt{\left(1 + j\omega\frac{L}{R}\right)\left(1 + j\omega\frac{C}{G}\right)}$$

For distortionless condition:

$$\frac{R}{L} = \frac{G}{C}$$

Therefore,

$$\frac{L}{R} = \frac{C}{G}$$

Hence,

$$\gamma = \sqrt{RG}\left(1 + j\omega\frac{L}{R}\right)$$

$$\gamma = \sqrt{RG} + j\omega\sqrt{RG}\frac{L}{R}$$

Now,

$$\sqrt{RG}\frac{L}{R} = \sqrt{LC}$$

Therefore,

$$\boxed{\gamma = \sqrt{RG} + j\omega\sqrt{LC}}$$

Since:

$$\gamma = \alpha + j\beta$$

Therefore,

$$\boxed{\alpha = \sqrt{RG}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

The attenuation constant is independent of frequency, and phase constant is directly proportional to frequency.

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Factor R and G :

$$Z_0 = \sqrt{\frac{R\left(1 + j\omega \frac{L}{R}\right)}{G\left(1 + j\omega \frac{C}{G}\right)}}$$

Using:

$$\frac{L}{R} = \frac{C}{G}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}}}$$

Also,

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Velocity of Wave

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

Hence for a distortionless line:

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

$$\boxed{\alpha = \sqrt{RG}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

Q8. Derive voltage and current wave equations in time domain for a transmission line.

Answer:

The solution of the voltage wave equation is:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

where,

$$\gamma = \alpha + j\beta$$

For a wave travelling in the $+z$ direction, consider only the incident wave:

$$V(z) = V_0 e^{-\gamma z}$$

Substitute:

$$\gamma = \alpha + j\beta$$

$$V(z) = V_0 e^{-(\alpha + j\beta)z}$$

$$V(z) = V_0 e^{-\alpha z} e^{-j\beta z}$$

Including the time factor:

$$e^{j\omega t}$$

The instantaneous voltage is:

$$V(z,t) = \text{Re}\left[V_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]$$

$$V(z,t) = \text{Re}\left[V_0 e^{-\alpha z} e^{j(\omega t - \beta z)}\right]$$

Therefore,

$$\boxed{V(z,t) = V_0 e^{-\alpha z} \cos(\omega t - \beta z)}$$

Similarly, the current wave is:

$$\boxed{I(z,t) = I_0 e^{-\alpha z} \cos(\omega t - \beta z)}$$

For a lossless line:

$$\alpha = 0$$

Therefore,

$$\boxed{V(z,t) = V_0 \cos(\omega t - \beta z)}$$

$$\boxed{I(z,t) = I_0 \cos(\omega t - \beta z)}$$

The term:

$$\omega t - \beta z$$

is the phase of the wave.

For a constant phase:

$$\omega t - \beta z = \text{constant}$$

Differentiating with respect to time:

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

Therefore, the phase velocity is:

$$\boxed{v_p = \frac{\omega}{\beta}}$$

Q9. Define phase velocity and group velocity. Derive their expressions and relation.

Answer:

Phase Velocity

Phase velocity is the velocity with which a wave of single frequency propagates along a transmission line.

For a wave:

$$V(z,t) = V_0 \cos(\omega t - \beta z)$$

For constant phase:

$$\omega t - \beta z = \text{constant}$$

Differentiating:

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

Therefore,

$$\boxed{v_p = \frac{\omega}{\beta}}$$

Group Velocity

Group velocity is the velocity with which the envelope of a complex wave propagates along the line. It is the ratio of change in angular frequency to change in phase constant.

$$\boxed{v_g = \frac{d\omega}{d\beta}}$$

Consider two harmonic waves of nearly equal angular frequencies:

$$y_1 = V_0 \cos(\omega_1 t - \beta_1 z)$$

$$y_2 = V_0 \cos(\omega_2 t - \beta_2 z)$$

The resultant is:

$$y = y_1 + y_2$$

Using trigonometric identities:

$$y = 2V_0 \cos\left[\frac{(\omega_1 - \omega_2)t - (\beta_1 - \beta_2)z}{2}\right] \cos\left[\frac{(\omega_1 + \omega_2)t - (\beta_1 + \beta_2)z}{2}\right]$$

The first cosine term represents the envelope. The velocity of the envelope is:

$$v_g = \frac{\omega_1 - \omega_2}{\beta_1 - \beta_2}$$

For very small difference:

$$\boxed{v_g = \frac{d\omega}{d\beta}}$$

Relation Between v_p and v_g

Phase velocity is:

$$v_p = \frac{\omega}{\beta}$$

Therefore,

$$\omega = \beta v_p$$

Differentiate with respect to β:

$$\frac{d\omega}{d\beta} = v_p + \beta \frac{dv_p}{d\beta}$$

But,

$$v_g = \frac{d\omega}{d\beta}$$

Therefore,

$$v_g = v_p + \beta \frac{dv_p}{d\beta}$$

From the notes relation:

$$\boxed{v_p = v_g \left[1 - \beta \frac{dv_p}{d\omega}\right]}$$

If phase velocity is independent of frequency, then:

$$\frac{dv_p}{d\omega} = 0$$

Therefore,

$$\boxed{v_p = v_g}$$

Thus, for a non-dispersive line, phase velocity and group velocity are equal.

Q10. Explain infinite transmission line and derive its input impedance.

Answer:

An infinite transmission line is a line of infinite length. Since the line is infinitely long, the incident wave never reaches the end of the line. Therefore, no reflected wave comes back.

For an infinite line:

$$V^- = 0$$

The voltage and current equations are:

$$V(z) = V_0^+ e^{-\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z}$$

The input impedance is:

$$Z_{in} = \frac{V(z)}{I(z)}$$

Substitute:

$$Z_{in} = \frac{V_0^+ e^{-\gamma z}}{\frac{V_0^+}{Z_0} e^{-\gamma z}}$$

$$\boxed{Z_{in} = Z_0}$$

Therefore, the input impedance of an infinite transmission line is equal to its characteristic impedance.

For a finite line, the general input impedance is:

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]}$$

For a lossless line:

$$\gamma = j\beta$$

and,

$$\tanh(j\beta l) = j \tan \beta l$$

Hence,

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]}$$

For an infinite line:

$$l \to \infty$$

The input impedance becomes:

$$\boxed{Z_{in} = Z_0}$$

Thus an infinite line behaves like a resistance equal to the characteristic impedance of the line.

Q11. Derive the input impedance relation of a transmission line terminated by load Z_L .

Answer:

Consider a transmission line of length l , characteristic impedance Z_0 , propagation constant γ , and load impedance Z_L .

The voltage and current waves are:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

At the load end:

$$Z_L = \frac{V_L}{I_L}$$

Using the wave equations and simplifying, the input impedance at a distance l from the load is:

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]}$$

This is the general input impedance of a lossy transmission line.

For Lossless Line

For a lossless line:

$$\alpha = 0$$

$$\gamma = j\beta$$

Therefore,

$$\tanh \gamma l = \tanh j\beta l$$

Using:

$$\tanh jx = j \tan x$$

we get:

$$\tanh j\beta l = j \tan \beta l$$

Hence,

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]}$$

This is the input impedance of a lossless transmission line.

Special Cases

If the load is matched:

$$Z_L = Z_0$$

Then:

$$Z_{in} = Z_0 \left[\frac{Z_0 + jZ_0 \tan \beta l}{Z_0 + jZ_0 \tan \beta l} \right]$$

$$\boxed{Z_{in} = Z_0}$$

Thus, when the line is matched, the input impedance is independent of length.

Q12. Derive input impedance of short-circuited and open-circuited lossless transmission lines.

Answer:

For a lossless transmission line:

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]}$$

Short-Circuited Line

For a short-circuited line:

$$Z_L = 0$$

Substitute in the input impedance formula:

$$Z_{sc} = Z_0 \left[\frac{0 + jZ_0 \tan \beta l}{Z_0 + j(0) \tan \beta l} \right]$$

$$Z_{sc} = Z_0 \left[\frac{jZ_0 \tan \beta l}{Z_0} \right]$$

$$\boxed{Z_{sc} = jZ_0 \tan \beta l}$$

Open-Circuited Line

For an open-circuited line:

$$Z_L \to \infty$$

The input impedance becomes:

$$Z_{oc} = \lim_{Z_L \to \infty} Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Dividing numerator and denominator by Z_L :

$$Z_{oc} = Z_0 \left[\frac{1 + j\frac{Z_0}{Z_L} \tan \beta l}{\frac{Z_0}{Z_L} + j \tan \beta l} \right]$$

As,

$$Z_L \to \infty$$

$$\frac{Z_0}{Z_L} \to 0$$

Therefore,

$$Z_{oc} = Z_0 \left[\frac{1}{j \tan \beta l} \right]$$

$$Z_{oc} = \frac{Z_0}{j \tan \beta l}$$

Since,

$$\frac{1}{j} = -j$$

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

Important Relation

$$Z_{sc} = jZ_0 \tan \beta l$$

$$Z_{oc} = -jZ_0 \cot \beta l$$

Multiplying:

$$Z_{sc} Z_{oc} = (jZ_0 \tan \beta l)(-jZ_0 \cot \beta l)$$

$$= Z_0^2 \tan \beta l \cot \beta l$$

$$\boxed{Z_{sc} Z_{oc} = Z_0^2}$$

Q13. Explain the behavior of short-circuited and open-circuited lines as circuit elements.

Answer:

Transmission lines can behave as inductors, capacitors, series resonant circuits, or parallel resonant circuits depending on their length and termination.

Short-Circuited Line

For a short-circuited line:

$$\boxed{Z_{sc} = jZ_0 \tan \beta l}$$

Case 1: $0 < l < \frac{\lambda}{4}$

For:

$$0 < \beta l < \frac{\pi}{2}$$

$$\tan \beta l > 0$$

Hence,

$$Z_{sc} = jX$$

This is inductive.

If the equivalent inductance is L_{eq} , then:

$$j\omega L_{eq} = jZ_0 \tan \beta l$$

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \tan \beta l}$$

Therefore, a short-circuited line of length less than $\lambda/4$ acts as an inductor.

Case 2: $\frac{\lambda}{4} < l < \frac{\lambda}{2}$

In this region:

$$\tan \beta l < 0$$

Therefore,

$$Z_{sc} = -jX$$

This is capacitive.

If equivalent capacitance is C_{eq} :

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan \beta l$$

$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \tan \beta l}}$$

Magnitude is considered for physical capacitance.

Case 3: $l = \frac{\lambda}{4}$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\tan \frac{\pi}{2} \to \infty$$

Thus,

$$Z_{sc} \to \infty$$

So a short-circuited line at $\lambda/4$ has infinite input impedance.

It acts as a parallel resonant or anti-resonant circuit.

Case 4: $l = \frac{\lambda}{2}$

$$\beta l = \pi$$

$$\tan \pi = 0$$

Therefore,

$$Z_{sc} = 0$$

So a short-circuited line at $\lambda/2$ has zero input impedance.

It acts as a series resonant circuit.

Open-Circuited Line

For an open-circuited line:

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

Case 1: $0 < l < \frac{\lambda}{4}$

For:

$$0 < \beta l < \frac{\pi}{2}$$

$$\cot \beta l > 0$$

Therefore:

$$Z_{oc} = -jX$$

This is capacitive.

If equivalent capacitance is C_{eq} :

$$\frac{1}{j\omega C_{eq}} = -jZ_0 \cot \beta l$$

$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}}$$

Thus an open-circuited line less than $\lambda/4$ acts as a capacitor.

Case 2: $\frac{\lambda}{4} < l < \frac{\lambda}{2}$

In this region:

$$\cot \beta l < 0$$

Thus:

$$Z_{oc} = +jX$$

It acts as an inductor.

If equivalent inductance is L_{eq} :

$$j\omega L_{eq} = jZ_0 \cot \beta l$$

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \cot \beta l}$$

Magnitude is considered for physical inductance.

Case 3: $l = \frac{\lambda}{4}$

$$\beta l = \frac{\pi}{2}$$

$$\cot \frac{\pi}{2} = 0$$

Therefore,

$$Z_{oc} = 0$$

So an open-circuited line at $\lambda/4$ has zero input impedance.

It acts as a series resonant circuit.

Case 4: $l = \frac{\lambda}{2}$

$$\beta l = \pi$$

$$\cot \pi \rightarrow \infty$$

Therefore,

$$Z_{oc} \rightarrow \infty$$

So an open-circuited line at $\lambda/2$ has infinite input impedance.

It acts as a parallel resonant or anti-resonant circuit.

Q14. Derive reflection coefficient and voltage standing wave ratio.

Answer:

When a transmission line is terminated by a load impedance different from its characteristic impedance, the incident wave is partly reflected from the load.

The voltage reflection coefficient is defined as:

$$\boxed{\Gamma = \frac{V_{0r}}{V_{0i}}}$$

where,

$$V_{0i} = \text{incident voltage amplitude}$$

$$V_{0r} = \text{reflected voltage amplitude}$$

At the load end:

$$Z_L = \frac{V_L}{I_L}$$

The total voltage at the load is:

$$V_L = V_{0i} + V_{0r}$$

The total current at the load is:

$$I_L = \frac{V_{0i}}{Z_0} - \frac{V_{0r}}{Z_0}$$

Therefore,

$$Z_L = \frac{V_{0i} + V_{0r}}{\frac{V_{0i}}{Z_0} - \frac{V_{0r}}{Z_0}}$$

$$Z_L = Z_0 \frac{V_{0i} + V_{0r}}{V_{0i} - V_{0r}}$$

Divide numerator and denominator by V_{0i} :

$$Z_L = Z_0 \frac{1 + \frac{V_{0r}}{V_{0i}}}{1 - \frac{V_{0r}}{V_{0i}}}$$

But,

$$\Gamma = \frac{V_{0r}}{V_{0i}}$$

Therefore,

$$Z_L = Z_0 \frac{1 + \Gamma}{1 - \Gamma}$$

Rearranging:

$$Z_L(1 - \Gamma) = Z_0(1 + \Gamma)$$

$$Z_L - Z_L\Gamma = Z_0 + Z_0\Gamma$$

$$Z_L - Z_0 = \Gamma(Z_L + Z_0)$$

Hence,

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Voltage Standing Wave Ratio

Due to interference between incident and reflected waves, standing waves are formed on the line.

Voltage Standing Wave Ratio is defined as:

$$\boxed{S = \frac{V_{max}}{V_{min}}}$$

For a lossless line:

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Let:

$$\Gamma = \frac{V_0^-}{V_0^+}$$

Then:

$$V_s(z) = V_0^+ e^{-j\beta z}\left[1 + \Gamma e^{2j\beta z}\right]$$

Maximum voltage occurs when incident and reflected voltages add:

$$V_{max} = V_0^+(1 + |\Gamma|)$$

Minimum voltage occurs when they subtract:

$$V_{min} = V_0^+(1 - |\Gamma|)$$

Therefore,

$$S = \frac{V_{max}}{V_{min}}$$

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

Special cases:

For matched load:

$$Z_L = Z_0$$

$$\Gamma = 0$$

$$S = 1$$

For complete reflection:

$$|\Gamma| = 1$$

$$S \to \infty$$

Q15. Derive the average power transmitted on a transmission line.

Answer:

The average power on a transmission line is:

$$P_{avg} = \frac{1}{T}\int_0^T V(z,t)I(z,t)\,dt$$

where T is the time period.

Let:

$$V(z,t) = V_0 e^{-\alpha z}\cos(\omega t - \beta z)$$

and

$$I(z,t) = I_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta)$$

Then,

$$P_{avg} = \frac{1}{T}\int_0^T V_0 e^{-\alpha z}\cos(\omega t - \beta z) I_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta)\,dt$$

$$P_{avg} = \frac{V_0 I_0 e^{-2\alpha z}}{T}\int_0^T \cos(\omega t - \beta z)\cos(\omega t - \beta z + \theta)\,dt$$

Using identity:

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

Here:

$$A = \omega t - \beta z$$

$$B = \omega t - \beta z + \theta$$

Therefore,

$$A - B = -\theta$$

$$A + B = 2\omega t - 2\beta z + \theta$$

So,

$$P_{avg} = \frac{V_0 I_0 e^{-2\alpha z}}{T} \int_0^T \frac{1}{2}[\cos\theta + \cos(2\omega t - 2\beta z + \theta)]dt$$

The average value of:

$$\cos(2\omega t - 2\beta z + \theta)$$

over one cycle is zero.

Hence,

$$P_{avg} = \frac{V_0 I_0 e^{-2\alpha z}}{2T} \int_0^T \cos\theta \, dt$$

$$P_{avg} = \frac{V_0 I_0 e^{-2\alpha z}}{2T} \cos\theta \int_0^T dt$$

$$P_{avg} = \frac{V_0 I_0 e^{-2\alpha z}}{2T} \cos\theta [T]$$

Therefore,

$$\boxed{P_{avg} = \frac{V_0 I_0}{2} e^{-2\alpha z} \cos\theta}$$

This shows that power decreases exponentially with distance when attenuation is present.

For a lossless line:

$$\alpha = 0$$

Therefore,

$$\boxed{P_{avg} = \frac{V_0 I_0}{2} \cos\theta}$$

Q16. Explain UHF lines as circuit elements.

Answer:

At ultra-high frequencies, ordinary lumped inductors and capacitors become difficult to use because the physical dimensions of circuit components become comparable to the wavelength. Therefore, short lengths of transmission lines are used as circuit elements.

For a lossless short-circuited line:

$$\boxed{Z_{sc} = jZ_0 \tan\beta l}$$

For a lossless open-circuited line:

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

Short-Circuited Line as Circuit Element

For $0 < l < \lambda/4$

$$Z_{sc} = jZ_0 \tan \beta l$$

Since:

$$\tan \beta l > 0$$

$$Z_{sc} = jX_L$$

Therefore, it acts as an inductor.

$$j\omega L_{eq} = jZ_0 \tan \beta l$$

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \tan \beta l}$$

For $\lambda/4 < l < \lambda/2$

$$\tan \beta l < 0$$

Therefore:

$$Z_{sc} = -jX_C$$

It acts as a capacitor.

$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \tan \beta l}}$$

Open-Circuited Line as Circuit Element

For $0 < l < \lambda/4$

$$Z_{oc} = -jZ_0 \cot \beta l$$

Since:

$$\cot \beta l > 0$$

$$Z_{oc} = -jX_C$$

It acts as a capacitor.

$$\frac{1}{j\omega C_{eq}} = -jZ_0 \cot \beta l$$

$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}}$$

For $\lambda/4 < l < \lambda/2$

$$\cot \beta l < 0$$

Therefore:

$$Z_{oc} = jX_L$$

It acts as an inductor.

$$j\omega L_{eq} = jZ_0 \cot \beta l$$

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \cot \beta l}$$

Resonant Behavior

Short-Circuited Line

At:

$$l = \frac{\lambda}{4}$$

$$Z_{sc} \to \infty$$

It acts as a parallel resonant circuit.

At:

$$l = \frac{\lambda}{2}$$

$$Z_{sc} = 0$$

It acts as a series resonant circuit.

Open-Circuited Line

At:

$$l = \frac{\lambda}{4}$$

$$Z_{oc} = 0$$

It acts as a series resonant circuit.

At:

$$l = \frac{\lambda}{2}$$

$$Z_{oc} \to \infty$$

It acts as a parallel resonant circuit.

Thus, UHF transmission lines are used as inductors, capacitors, and resonant circuits.

Q17. Write a 10-mark answer on transients on transmission lines.

Answer:

A transient on a transmission line occurs when a sudden change in voltage or current is applied to the line, such as switching ON a source or applying a pulse. During the transient condition, voltage and current waves travel along the line with finite velocity.

For a transmission line, the travelling voltage wave is:

$$V^{+}(z, t)$$

and the reflected voltage wave is:

$$V^{-}(z, t)$$

The corresponding currents are:

$$I^+(z,t) = \frac{V^+(z,t)}{Z_0}$$

$$I^-(z,t) = -\frac{V^-(z,t)}{Z_0}$$

The negative sign appears because the reflected current travels in the opposite direction.

Load Reflection Coefficient

At the load end, if the load impedance is Z_L , the voltage reflection coefficient is:

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

The reflected voltage at the load is:

$$\boxed{V^- = \Gamma_L V^+}$$

Source Reflection Coefficient

At the source end, if the source impedance is Z_g , the source reflection coefficient is:

$$\boxed{\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}}$$

When the reflected wave reaches the source, it may again be reflected depending on Z_g .

Special Cases

For matched load:

$$Z_L = Z_0$$

$$\Gamma_L = 0$$

There is no reflection.

For short-circuited load:

$$Z_L = 0$$

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0}$$

$$\boxed{\Gamma_L = -1}$$

Voltage reflects with reversal.

For open-circuited load:

$$Z_L \to \infty$$

$$\boxed{\Gamma_L = +1}$$

Voltage reflects without reversal.

Travelling Time

If the length of the line is l and velocity is v , then the time taken by a wave to reach the load is:

$$\boxed{t_d = \frac{l}{v}}$$

The round-trip time is:

$$\boxed{2t_d = \frac{2l}{v}}$$

Transient Build-Up

If a voltage step is applied to the line, an incident voltage wave travels towards the load. At the load, a part of the wave is absorbed and a part is reflected. The reflected wave travels back to the source. At the source end, it is again reflected depending on source impedance. This process continues until the steady-state voltage and current distribution is reached.

The voltage at any point on the line is the sum of the incident and reflected waves:

$$\boxed{V = V^+ + V^-}$$

The current at any point is:

$$\boxed{I = \frac{V^+}{Z_0} - \frac{V^-}{Z_0}}$$

Thus, transients on transmission lines are explained using travelling waves and reflection coefficients.

IMPORTANT FORMULA LIST FOR QUICK REVISION

$$\boxed{-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}}$$

$$\boxed{-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}}$$

$$\boxed{-\frac{dV}{dz} = (R + j\omega L)I}$$

$$\boxed{-\frac{dI}{dz} = (G + j\omega C)V}$$

$$\boxed{\frac{d^2V}{dz^2} - \gamma^2 V = 0}$$

$$\boxed{\frac{d^2I}{dz^2} - \gamma^2 I = 0}$$

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\boxed{\gamma = \alpha + j\beta}$$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

$$\boxed{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}}$$

$$\boxed{I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}}$$

$$\boxed{V(z,t) = V_0 e^{-\alpha z}\cos(\omega t - \beta z)}$$

$$\boxed{I(z,t) = I_0 e^{-\alpha z}\cos(\omega t - \beta z)}$$

$$\boxed{v_p = \frac{\omega}{\beta}}$$

$$\boxed{v_g = \frac{d\omega}{d\beta}}$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]}$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]}$$

$$\boxed{Z_{sc} = jZ_0 \tan \beta l}$$

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

$$\boxed{Z_{sc} Z_{oc} = Z_0^2}$$

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

$$\boxed{P_{avg} = \frac{V_0 I_0}{2} e^{-2\alpha z} \cos \theta}$$

NUMERICAL PROBLEMS / SUMS

Problem 1

A distortionless line has:

$$Z_0 = 60\Omega$$

$$\alpha = 20\, mNp/m = 20 \times 10^{-3} Np/m$$

Velocity:

$$u = 0.6c$$

where,

$$c = 3 \times 10^8 m/s$$

Find:

$$R,\ L,\ G,\ C,\ \lambda$$

at:

$$f = 100MHz$$

Solution:

For a distortionless line:

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

Given:

$$Z_0 = 60\Omega$$

Therefore,

$$\sqrt{\frac{R}{G}} = 60$$

$$\frac{R}{G} = 60^2$$

$$R = 60^2 G$$

Also,

$$\alpha = \sqrt{RG}$$

Given:

$$\alpha = 20 \times 10^{-3} Np/m$$

$$\sqrt{RG} = 20 \times 10^{-3}$$

Substitute:

$$R = 60^2 G$$

$$\sqrt{60^2 G^2} = 20 \times 10^{-3}$$

$$60G = 20 \times 10^{-3}$$

$$G = \frac{20 \times 10^{-3}}{60}$$

$$\boxed{G = 3.33 \times 10^{-4} S/m}$$

Now,

$$R = 60^2 G$$

$$R = 3600(3.33 \times 10^{-4})$$

$$\boxed{R = 1.2 \Omega/m}$$

Velocity:

$$u = \frac{1}{\sqrt{LC}}$$

Given:

$$u = 0.6c$$

$$u = 0.6 \times 3 \times 10^8$$

$$u = 1.8 \times 10^8 m/s$$

Also:

$$Z_0 = \sqrt{\frac{L}{C}} = 60$$

$$L = 60^2 C$$

Now:

$$u = \frac{1}{\sqrt{LC}}$$

$$LC = \frac{1}{u^2}$$

Substitute:

$$L = 60^2 C$$

$$60^2 C^2 = \frac{1}{u^2}$$

$$60C = \frac{1}{u}$$

$$C = \frac{1}{60u}$$

$$C = \frac{1}{60(1.8 \times 10^8)}$$

$$\boxed{C = 92.59pF/m}$$

Now:

$$L = 60^2 C$$

$$L = 3600(92.59 \times 10^{-12})$$

$$\boxed{L = 333.3nH/m}$$

Wavelength:

$$u = f\lambda$$

$$\lambda = \frac{u}{f}$$

$$\lambda = \frac{1.8 \times 10^8}{100 \times 10^6}$$

$$\boxed{\lambda = 1.8m}$$

Final answers:

$$\boxed{R = 1.2\Omega/m}$$

$$\boxed{G = 3.33 \times 10^{-4} S/m}$$

$$\boxed{C = 92.59pF/m}$$

$$\boxed{L = 333.3nH/m}$$

$$\boxed{\lambda = 1.8m}$$

Problem 2

An air line has:

$$Z_0 = 70\Omega$$

$$\beta = 3rad/m$$

at:

$$f = 100MHz$$

Calculate:

$$L/m, \quad C/m$$

Solution:

For air line:

$$R = 0, \qquad G = 0$$

Therefore, it is lossless.

For a lossless line:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Given:

$$Z_0 = 70\Omega$$

$$\sqrt{\frac{L}{C}} = 70$$

$$\frac{L}{C} = 70^2$$

$$L = 70^2 C$$

Also,

$$\beta = \omega\sqrt{LC}$$

Given:

$$\beta = 3rad/m$$

$$\omega = 2\pi f$$

$$\omega = 2\pi(100 \times 10^6)$$

$$\omega = 2\pi \times 10^8$$

Now:

$$\sqrt{LC} = \frac{\beta}{\omega}$$

Multiplying:

$$Z_0\beta = \sqrt{\frac{L}{C}}\omega\sqrt{LC}$$

$$Z_0\beta = \omega L$$

Therefore:

$$L = \frac{Z_0\beta}{\omega}$$

$$L = \frac{70 \times 3}{2\pi \times 100 \times 10^6}$$

$$L = \frac{210}{2\pi \times 10^8}$$

$$\boxed{L = 334.22nH/m}$$

Now:

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$C = \frac{L}{Z_0^2}$$

$$C = \frac{334.22 \times 10^{-9}}{70^2}$$

$$C = \frac{334.22 \times 10^{-9}}{4900}$$

$$\boxed{C = 68.2pF/m}$$

Final answers:

$$\boxed{L = 334.22nH/m}$$

$$\boxed{C = 68.2pF/m}$$

Problem 3

A certain transmission line operating at:

$$\omega = 10^6 rad/s$$

has:

$$\alpha = 8dB/m$$

$$\beta = 1rad/m$$

$$Z_0 = 60 + j40\Omega$$

Length:

$$l = 2m$$

The line is connected to a source:

$$V_g = 10\angle 0^\circ V$$

$$Z_g = 40\Omega$$

and terminated by load:

$$Z_L = 20 + j50\Omega$$

Determine:

1. Input impedance
2. Sending end current

Solution:

Given attenuation is in dB/m. Convert into Np/m:

$$1Np = 8.686dB$$

$$\alpha = \frac{8}{8.686}$$

$$\boxed{\alpha = 0.921Np/m}$$

Propagation constant:

$$\gamma = \alpha + j\beta$$

$$\gamma = 0.921 + j1$$

Length:

$$l = 2m$$

$$\gamma l = 2(0.921 + j1)$$

$$\gamma l = 1.842 + j2$$

From notes:

$$\tanh(\gamma l) = 1.033 - j0.03929$$

The input impedance of a lossy line is:

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

Substitute:

$$Z_0 = 60 + j40$$

$$Z_L = 20 + j50$$

$$\tanh \gamma l = 1.033 - j0.03929$$

$$Z_{in} = (60 + j40) \left[\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)} \right]$$

After simplification:

$$\boxed{Z_{in} = 60.25 + j38.79\Omega}$$

Sending-End Current

The sending-end current is:

$$I(z = 0) = I_0$$

The source circuit contains:

$$Z_g$$

in series with:

$$Z_{in}$$

Therefore,

$$I_0 = \frac{V_g}{Z_g + Z_{in}}$$

$$I_0 = \frac{10\angle 0^\circ}{40 + (60.25 + j38.79)}$$

$$I_0 = \frac{10}{100.25 + j38.79}$$

Magnitude of denominator:

$$|100.25 + j38.79| = \sqrt{100.25^2 + 38.79^2}$$

$$|100.25 + j38.79| = 107.49$$

Angle:

$$\theta = \tan^{-1}\left(\frac{38.79}{100.25}\right)$$

$$\theta = 21.15^\circ$$

Therefore:

$$I_0 = \frac{10\angle 0^\circ}{107.49\angle 21.15^\circ}$$

$$I_0 = 0.09303\angle(-21.15^\circ)A$$

$$\boxed{I_0 = 93.03\angle - 21.15^\circ mA}$$

Final answers:

$$\boxed{Z_{in} = 60.25 + j38.79\Omega}$$

$$\boxed{I_0 = 93.03\angle - 21.15^\circ mA}$$

Problem 4

A lossless transmission line has:

$$Z_0 = 50\Omega$$

It is terminated by:

$$Z_L = 100\Omega$$

Find the reflection coefficient and VSWR.

Solution:

Reflection coefficient:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{100 - 50}{100 + 50}$$

$$\Gamma = \frac{50}{150}$$

$$\boxed{\Gamma = \frac{1}{3} = 0.333}$$

VSWR:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.333}{1 - 0.333}$$

$$S = \frac{1.333}{0.667}$$

$$\boxed{S = 2}$$

Problem 5

A short-circuited lossless line has:

$$Z_0 = 75\Omega$$

$$l = \frac{\lambda}{8}$$

Find input impedance.

Solution:

For a short-circuited line:

$$Z_{sc} = jZ_0 \tan \beta l$$

Now:

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}$$

$$\beta l = \frac{\pi}{4}$$

Therefore:

$$Z_{sc} = j75 \tan \frac{\pi}{4}$$

$$\tan \frac{\pi}{4} = 1$$

$$\boxed{Z_{sc} = j75\Omega}$$

The line acts as an inductor.

Problem 6

An open-circuited lossless line has:

$$Z_0 = 60\Omega$$

$$l = \frac{\lambda}{8}$$

Find input impedance.

Solution:

For open-circuited line:

$$Z_{oc} = -jZ_0 \cot \beta l$$

Now:

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}$$

$$\beta l = \frac{\pi}{4}$$

Therefore:

$$Z_{oc} = -j60 \cot \frac{\pi}{4}$$

$$\cot \frac{\pi}{4} = 1$$

$$\boxed{Z_{oc} = -j60\Omega}$$

The line acts as a capacitor.

Problem 7

For a lossless line:

$$L = 250nH/m$$

$$C = 100pF/m$$

Find:

$$Z_0, \quad v, \quad \beta$$

at:

$$f = 100MHz$$

Solution:

Characteristic impedance:

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{250 \times 10^{-9}}{100 \times 10^{-12}}}$$

$$Z_0 = \sqrt{2500}$$

$$\boxed{Z_0 = 50\Omega}$$

Velocity:

$$v = \frac{1}{\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{(250 \times 10^{-9})(100 \times 10^{-12})}}$$

$$v = \frac{1}{\sqrt{25 \times 10^{-18}}}$$

$$v = \frac{1}{5 \times 10^{-9}}$$

$$\boxed{v = 2 \times 10^{8} m/s}$$

Angular frequency:

$$\omega = 2\pi f$$

$$\omega = 2\pi(100 \times 10^{6})$$

$$\omega = 2\pi \times 10^{8}$$

Phase constant:

$$\beta = \frac{\omega}{v}$$

$$\beta = \frac{2\pi \times 10^{8}}{2 \times 10^{8}}$$

$$\boxed{\beta = \pi rad/m}$$

Problem 8

A line has:

$$Z_0 = 50\Omega$$

and the load is:

$$Z_L = 25\Omega$$

Find reflection coefficient and VSWR.

Solution:

Reflection coefficient is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substitute values:

$$\Gamma = \frac{25 - 50}{25 + 50}$$

$$\Gamma = \frac{-25}{75}$$

$$\boxed{\Gamma = -0.333}$$

Magnitude of reflection coefficient:

$$|\Gamma| = 0.333$$

VSWR is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.333}{1 - 0.333}$$

$$S = \frac{1.333}{0.667}$$

$$\boxed{S = 2}$$

Therefore:

$$\boxed{\Gamma = -0.333}$$

$$\boxed{VSWR = 2}$$

The negative sign of Γ shows that the reflected voltage wave undergoes phase reversal.

Problem 9

A lossless transmission line has:

$$Z_0 = 100\Omega$$

and is terminated by:

$$Z_L = 100\Omega$$

Find the reflection coefficient and VSWR.

Solution:

Reflection coefficient is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substitute values:

$$\Gamma = \frac{100 - 100}{100 + 100}$$

$$\Gamma = \frac{0}{200}$$

$$\boxed{\Gamma = 0}$$

VSWR is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0}{1 - 0}$$

$$\boxed{S = 1}$$

Therefore, the line is perfectly matched.

$$\boxed{Z_L = Z_0}$$

Hence there is no reflection.

Problem 10

A lossless transmission line has:

$$Z_0 = 50\Omega$$

and is terminated in a short circuit. Find the reflection coefficient.

Solution:

For a short-circuited line:

$$Z_L = 0$$

Reflection coefficient is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substitute:

$$\Gamma = \frac{0 - 50}{0 + 50}$$

$$\Gamma = \frac{-50}{50}$$

$$\boxed{\Gamma = -1}$$

For a short circuit, the magnitude is:

$$|\Gamma| = 1$$

Therefore, complete reflection occurs.

The negative sign indicates that the reflected voltage wave is reversed in phase.

VSWR is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 1}{1 - 1}$$

$$S = \frac{2}{0}$$

$$\boxed{S = \infty}$$

Therefore, a short-circuited transmission line produces complete reflection and infinite VSWR.

Problem 11

A lossless transmission line has:

$$Z_0 = 50\Omega$$

and is terminated in an open circuit. Find reflection coefficient and VSWR.

Solution:

For an open-circuited line:

$$Z_L \to \infty$$

Reflection coefficient is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Dividing numerator and denominator by Z_L :

$$\Gamma = \frac{1 - \frac{Z_0}{Z_L}}{1 + \frac{Z_0}{Z_L}}$$

As:

$$Z_L \to \infty$$

$$\frac{Z_0}{Z_L} \to 0$$

Therefore:

$$\Gamma = \frac{1 - 0}{1 + 0}$$

$$\boxed{\Gamma = +1}$$

Magnitude:

$$|\Gamma| = 1$$

VSWR is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 1}{1 - 1}$$

$$S = \frac{2}{0}$$

$$\boxed{S = \infty}$$

Thus, an open-circuited line gives complete reflection without phase reversal of voltage.

Problem 12

A short-circuited lossless line has:

$$Z_0 = 100\Omega$$

and length:

$$l = \frac{\lambda}{4}$$

Find the input impedance.

Solution:

For a short-circuited lossless transmission line:

$$Z_{sc} = jZ_0 \tan \beta l$$

Now,

$$\beta = \frac{2\pi}{\lambda}$$

Given:

$$l = \frac{\lambda}{4}$$

Therefore:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\beta l = \frac{\pi}{2}$$

So:

$$Z_{sc} = jZ_0 \tan \frac{\pi}{2}$$

But:

$$\tan \frac{\pi}{2} \to \infty$$

Therefore:

$$Z_{sc} \to \infty$$

$$\boxed{Z_{in} = \infty}$$

Hence, a short-circuited line of length:

$$\frac{\lambda}{4}$$

acts as an open circuit.

It behaves as a parallel resonant or anti-resonant circuit.

Problem 13

An open-circuited lossless line has:

$$Z_0 = 100\Omega$$

and length:

$$l = \frac{\lambda}{4}$$

Find the input impedance.

Solution:

For an open-circuited lossless line:

$$Z_{oc} = -jZ_0 \cot \beta l$$

Given:

$$l = \frac{\lambda}{4}$$

Now:

$$\beta = \frac{2\pi}{\lambda}$$

Therefore:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\beta l = \frac{\pi}{2}$$

So:

$$Z_{oc} = -jZ_0 \cot \frac{\pi}{2}$$

But:

$$\cot \frac{\pi}{2} = 0$$

Therefore:

$$Z_{oc} = 0$$

$$\boxed{Z_{in} = 0}$$

Hence, an open-circuited line of length:

$$\frac{\lambda}{4}$$

acts as a short circuit.

It behaves as a series resonant circuit.

Problem 14

A short-circuited lossless transmission line has:

$$Z_0 = 60\Omega$$

and length:

$$l = \frac{\lambda}{2}$$

Find the input impedance.

Solution:

For short-circuited line:

$$Z_{sc} = jZ_0 \tan \beta l$$

Given:

$$l = \frac{\lambda}{2}$$

Now:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$\beta l = \pi$$

Therefore:

$$Z_{sc} = j60 \tan \pi$$

But:

$$\tan \pi = 0$$

So:

$$Z_{sc} = 0$$

$$\boxed{Z_{in} = 0}$$

Thus, a short-circuited line of length:

$$\frac{\lambda}{2}$$

acts as a short circuit at the input.

It behaves as a series resonant circuit.

Problem 15

An open-circuited lossless transmission line has:

$$Z_0 = 60\Omega$$

and length:

$$l = \frac{\lambda}{2}$$

Find the input impedance.

Solution:

For open-circuited line:

$$Z_{oc} = -jZ_0 \cot \beta l$$

Given:

$$l = \frac{\lambda}{2}$$

Now:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$\beta l = \pi$$

Therefore:

$$Z_{oc} = -j60 \cot \pi$$

But:

$$\cot \pi \to \infty$$

Therefore:

$$Z_{oc} \to \infty$$

$$\boxed{Z_{in} = \infty}$$

Thus, an open-circuited line of length:

$$\frac{\lambda}{2}$$

acts as an open circuit at the input.

It behaves as a parallel resonant or anti-resonant circuit.

Problem 16

A lossless line has:

$$Z_0 = 75\Omega$$

The length of the line is:

$$l = \frac{\lambda}{8}$$

Find the input impedance when the line is short-circuited and when it is open-circuited.

Solution:

Given:

$$Z_0 = 75\Omega$$

$$l = \frac{\lambda}{8}$$

Now:

$$\beta = \frac{2\pi}{\lambda}$$

Therefore:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8}$$

$$\beta l = \frac{\pi}{4}$$

Case 1: Short-Circuited Line

For short-circuited line:

$$Z_{sc} = jZ_0 \tan \beta l$$

$$Z_{sc} = j75 \tan \frac{\pi}{4}$$

Since:

$$\tan \frac{\pi}{4} = 1$$

$$\boxed{Z_{sc} = j75\Omega}$$

Hence, the short-circuited line acts as an inductor.

Case 2: Open-Circuited Line

For open-circuited line:

$$Z_{oc} = -jZ_0 \cot \beta l$$

$$Z_{oc} = -j75 \cot \frac{\pi}{4}$$

Since:

$$\cot \frac{\pi}{4} = 1$$

$$\boxed{Z_{oc} = -j75\Omega}$$

Hence, the open-circuited line acts as a capacitor.

Problem 17

A short-circuited lossless line has:

$$Z_0 = 50\Omega$$

and:

$$\beta l = 30^\circ$$

Find the equivalent inductance at:

$$f = 100MHz$$

Solution:

For a short-circuited line of length less than:

$$\frac{\lambda}{4}$$

it acts as an inductor.

Input impedance is:

$$Z_{sc} = jZ_0 \tan \beta l$$

Equivalent inductive reactance is:

$$j\omega L_{eq} = jZ_0 \tan \beta l$$

Therefore:

$$L_{eq} = \frac{Z_0}{\omega} \tan \beta l$$

Given:

$$Z_0 = 50\Omega$$

$$\beta l = 30^\circ$$

$$f = 100MHz$$

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 100 \times 10^6$$

$$\omega = 2\pi \times 10^8$$

Now:

$$L_{eq} = \frac{50}{2\pi \times 10^8} \tan 30^\circ$$

$$\tan 30^\circ = 0.577$$

$$L_{eq} = \frac{50 \times 0.577}{2\pi \times 10^8}$$

$$L_{eq} = \frac{28.85}{6.283 \times 10^8}$$

$$L_{eq} = 4.59 \times 10^{-8} H$$

$$\boxed{L_{eq} = 45.9nH}$$

Problem 18

An open-circuited lossless line has:

$$Z_0 = 50\Omega$$

and:

$$\beta l = 30^\circ$$

Find the equivalent capacitance at:

$$f = 100MHz$$

Solution:

For an open-circuited line of length less than:

$$\frac{\lambda}{4}$$

it acts as a capacitor.

Input impedance is:

$$Z_{oc} = -jZ_0 \cot \beta l$$

Equivalent capacitive impedance is:

$$Z_C = \frac{1}{j\omega C_{eq}}$$

But:

$$\frac{1}{j\omega C_{eq}} = -jZ_0 \cot \beta l$$

Therefore:

$$C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}$$

Given:

$$Z_0 = 50\Omega$$

$$\beta l = 30^\circ$$

$$f = 100MHz$$

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 100 \times 10^6$$

$$\omega = 2\pi \times 10^8$$

Now:

$$C_{eq} = \frac{1}{(2\pi \times 10^8)(50) \cot 30^\circ}$$

$$\cot 30^\circ = 1.732$$

$$C_{eq} = \frac{1}{(6.283 \times 10^8)(50)(1.732)}$$

$$C_{eq} = \frac{1}{5.441 \times 10^{10}}$$

$$C_{eq} = 1.838 \times 10^{-11} F$$

$$\boxed{C_{eq} = 18.38pF}$$

Problem 19

A transmission line has:

$$R = 1\Omega/m$$

$$L = 250nH/m$$

$$G = 4 \times 10^{-4}S/m$$

$$C = 100pF/m$$

Check whether the line is distortionless.

Solution:

For a distortionless line, the condition is:

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

or:

$$\boxed{RC = LG}$$

Given:

$$R = 1\Omega/m$$

$$L = 250 \times 10^{-9}H/m$$

$$G = 4 \times 10^{-4}S/m$$

$$C = 100 \times 10^{-12}F/m$$

Calculate:

$$\frac{R}{L} = \frac{1}{250 \times 10^{-9}}$$
$$\frac{R}{L} = 4 \times 10^{6}$$

Now:

$$\frac{G}{C} = \frac{4 \times 10^{-4}}{100 \times 10^{-12}}$$

$$\frac{G}{C} = 4 \times 10^6$$

Since:

$$\frac{R}{L} = \frac{G}{C}$$

$$4 \times 10^6 = 4 \times 10^6$$

Therefore, the line is distortionless.

$$\boxed{\text{The given line is distortionless.}}$$

Problem 20

For a distortionless line:

$$R = 1.2\Omega/m$$

$$G = 3.33 \times 10^{-4} S/m$$

Find the attenuation constant.

Solution:

For a distortionless line:

$$\alpha = \sqrt{RG}$$

Substitute values:

$$\alpha = \sqrt{(1.2)(3.33 \times 10^{-4})}$$

$$\alpha = \sqrt{3.996 \times 10^{-4}}$$

$$\alpha = 1.998 \times 10^{-2}$$

$$\boxed{\alpha = 0.01998 Np/m}$$

Approximately:

$$\boxed{\alpha = 20 mNp/m}$$

Problem 21

For a lossless line:

$$L = 400nH/m$$

$$C = 100pF/m$$

Find:

$$Z_0, \quad v, \quad \beta$$

at:

$$f = 50MHz$$

Solution:

For a lossless line:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Substitute:

$$Z_0 = \sqrt{\frac{400 \times 10^{-9}}{100 \times 10^{-12}}}$$

$$Z_0 = \sqrt{4000}$$

$$\boxed{Z_0 = 63.25\Omega}$$

Velocity:

$$v = \frac{1}{\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{(400 \times 10^{-9})(100 \times 10^{-12})}}$$

$$v = \frac{1}{\sqrt{40 \times 10^{-18}}}$$

$$v = \frac{1}{6.324 \times 10^{-9}}$$

$$\boxed{v = 1.581 \times 10^8 m/s}$$

Angular frequency:

$$\omega = 2\pi f$$

$$\omega = 2\pi(50 \times 10^6)$$

$$\omega = 3.142 \times 10^8 rad/s$$

Phase constant:

$$\beta = \frac{\omega}{v}$$

$$\beta = \frac{3.142 \times 10^8}{1.581 \times 10^8}$$

$$\boxed{\beta = 1.987 rad/m}$$

Problem 22

A lossless line has:

$$Z_0 = 50\Omega$$

$$Z_L = 100\Omega$$

and length:

$$l = \frac{\lambda}{4}$$

Find the input impedance.

Solution:

For a lossless line:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Given:

$$l = \frac{\lambda}{4}$$

Therefore:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\beta l = \frac{\pi}{2}$$

At:

$$\beta l = \frac{\pi}{2}$$

$$\tan \beta l \to \infty$$

For a quarter-wave line, the input impedance becomes:

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Substitute:

$$Z_{in} = \frac{50^2}{100}$$

$$Z_{in} = \frac{2500}{100}$$

$$\boxed{Z_{in} = 25\Omega}$$

Problem 23

A lossless line has:

$$Z_0 = 75\Omega$$

$$Z_L = 25\Omega$$

and length:

$$l = \frac{\lambda}{4}$$

Find the input impedance.

Solution:

For a quarter-wave line:

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Substitute values:

$$Z_{in} = \frac{75^2}{25}$$

$$Z_{in} = \frac{5625}{25}$$

$$\boxed{Z_{in} = 225\Omega}$$

Thus, a quarter-wave line transforms a low impedance into a high impedance.

Problem 24

A lossless line has:

$$Z_0 = 50\Omega$$

$$Z_L = 100\Omega$$

and length:

$$l = \frac{\lambda}{2}$$

Find the input impedance.

Solution:

For a half-wave line:

$$l = \frac{\lambda}{2}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$\beta l = \pi$$

Since:

$$\tan \pi = 0$$

The lossless input impedance is:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Substitute:

$$\tan \beta l = 0$$

$$Z_{in} = Z_0 \left[\frac{Z_L}{Z_0} \right]$$

$$Z_{in} = Z_L$$

Therefore:

$$\boxed{Z_{in} = 100\Omega}$$

A half-wave line repeats the load impedance at the input.

Problem 25

A transmission line has:

$$V_{max} = 120V$$

and:

$$V_{min} = 40V$$

Find VSWR and reflection coefficient magnitude.

Solution:

VSWR is:

$$S = \frac{V_{max}}{V_{min}}$$

Substitute:

$$S = \frac{120}{40}$$

$$\boxed{S = 3}$$

Relation between VSWR and reflection coefficient is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Rearrange:

$$S(1 - |\Gamma|) = 1 + |\Gamma|$$

$$S - S|\Gamma| = 1 + |\Gamma|$$

$$S - 1 = |\Gamma|(S + 1)$$

$$|\Gamma| = \frac{S - 1}{S + 1}$$

Substitute:

$$|\Gamma| = \frac{3 - 1}{3 + 1}$$

$$|\Gamma| = \frac{2}{4}$$

$$\boxed{|\Gamma| = 0.5}$$

Problem 26

A transmission line has reflection coefficient magnitude:

$$|\Gamma| = 0.25$$

Find VSWR.

Solution:

VSWR is:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Substitute:

$$S = \frac{1 + 0.25}{1 - 0.25}$$

$$S = \frac{1.25}{0.75}$$

$$\boxed{S = 1.667}$$

Problem 27

A wave travels on a transmission line with:

$$\beta = 4 rad/m$$

and angular frequency:

$$\omega = 2 \times 10^8 rad/s$$

Find phase velocity.

Solution:

Phase velocity is:

$$v_p = \frac{\omega}{\beta}$$

Substitute:

$$v_p = \frac{2 \times 10^8}{4}$$

$$\boxed{v_p = 5 \times 10^7 m/s}$$

Problem 28

For a line:

$$\omega = 6 \times 10^8 rad/s$$

and:

$$v_p = 2 \times 10^8 m/s$$

Find phase constant β.

Solution:

Phase velocity is:

$$v_p = \frac{\omega}{\beta}$$

Therefore:

$$\beta = \frac{\omega}{v_p}$$

Substitute:

$$\beta = \frac{6 \times 10^8}{2 \times 10^8}$$

$$\boxed{\beta = 3rad/m}$$

Problem 29

A wave travels with velocity:

$$v = 1.5 \times 10^8 m/s$$

at frequency:

$$f = 75MHz$$

Find wavelength.

Solution:

Velocity, frequency and wavelength are related by:

$$v = f\lambda$$

Therefore:

$$\lambda = \frac{v}{f}$$

Substitute:

$$\lambda = \frac{1.5 \times 10^8}{75 \times 10^6}$$

$$\lambda = \frac{1.5 \times 10^8}{7.5 \times 10^7}$$

$$\boxed{\lambda = 2m}$$

Problem 30

A lossless transmission line has:

$$Z_0 = 80\Omega$$

and length:

$$l = \frac{\lambda}{8}$$

It is terminated by:

$$Z_L = 80\Omega$$

Find input impedance.

Solution:

Since:

$$Z_L = Z_0$$

the line is matched.

For a matched transmission line:

$$\boxed{Z_{in} = Z_0}$$

Therefore:

$$\boxed{Z_{in} = 80\Omega}$$

This result is independent of length.

Since the line is matched:

$$\Gamma = 0$$

and:

$$VSWR = 1$$

EXTRA EXAM-STYLE 10-MARK QUESTIONS WITHOUT FULL NUMERICALS

You can also prepare these as theory questions:

Q18. Explain the significance of characteristic impedance and propagation constant.

Answer:

The characteristic impedance Z_0 and propagation constant γ are the two most important secondary constants of a transmission line.

The characteristic impedance is:

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

It represents the ratio of voltage to current for a travelling wave on an infinitely long transmission line.

$$\boxed{Z_0 = \frac{V^+}{I^+}}$$

If a line is terminated with:

$$Z_L = Z_0$$

then no reflection occurs. Hence:

$$\Gamma = 0$$

and:

$$VSWR = 1$$

The propagation constant is:

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

Also:

$$\boxed{\gamma = \alpha + j\beta}$$

where:

$$\alpha = \text{attenuation constant}$$

$$\beta = \text{phase constant}$$

The attenuation constant shows the reduction of wave amplitude with distance:

$$V(z,t) = V_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

The phase constant determines phase change per unit length.

Thus, Z_0 determines matching and reflections, while γ determines attenuation and phase shift.

Q19. Explain reflection on transmission lines and mention special cases.

Answer:

When a travelling wave reaches the load end of a transmission line, if the load impedance is not equal to the characteristic impedance, then the load cannot absorb the entire incident power. A part of the wave is reflected back towards the source.

The voltage reflection coefficient is defined as:

$$\boxed{\Gamma = \frac{V_r}{V_i}}$$

At the load:

$$V_L = V_i + V_r$$

$$I_L = \frac{V_i}{Z_0} - \frac{V_r}{Z_0}$$

Therefore:

$$Z_L = \frac{V_i + V_r}{\frac{V_i}{Z_0} - \frac{V_r}{Z_0}}$$

$$Z_L = Z_0 \frac{V_i + V_r}{V_i - V_r}$$

Dividing by V_i :

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

Solving:

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Special cases:

Matched Line

$$Z_L = Z_0$$

$$\Gamma = 0$$

No reflection.

Short Circuit

$$Z_L = 0$$

$$\Gamma = -1$$

Complete reflection with voltage reversal.

Open Circuit

$$Z_L \to \infty$$

$$\Gamma = +1$$

Complete reflection without voltage reversal.

The VSWR is:

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

For matched line:

$$S = 1$$

For open or short circuit:

$$S = \infty$$

Q20. Write short notes on UHF transmission lines as inductors and capacitors.

Answer:

At UHF, ordinary lumped inductors and capacitors are not suitable because the wavelength is small and the physical size of components becomes comparable with the wavelength. Therefore, small sections of transmission lines are used as circuit elements.

For a short-circuited lossless line:

$$\boxed{Z_{sc} = jZ_0 \tan \beta l}$$

For:

$$0 < l < \frac{\lambda}{4}$$

$$\tan \beta l > 0$$

Therefore:

$$Z_{sc} = jX_L$$

So it acts as an inductor.

$$j\omega L_{eq} = jZ_0 \tan \beta l$$

$$\boxed{L_{eq} = \frac{Z_0}{\omega} \tan \beta l}$$

For:

$$\frac{\lambda}{4} < l < \frac{\lambda}{2}$$

$$\tan \beta l < 0$$

Therefore it acts as a capacitor.

For an open-circuited lossless line:

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

For:

$$0 < l < \frac{\lambda}{4}$$

$$\cot \beta l > 0$$

Therefore:

$$Z_{oc} = -jX_C$$

So it acts as a capacitor.

$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}}$$

For:

$$\frac{\lambda}{4} < l < \frac{\lambda}{2}$$

it acts as an inductor.

Thus, short and open-circuited transmission lines can be used as inductors, capacitors, and resonant circuits at UHF.

FINAL QUICK EXAM REVISION TABLE

Case	Input Impedance	Behavior
Short line	$Z_{sc} = jZ_0 \tan \beta l$	Depends on length
Open line	$Z_{oc} = -jZ_0 \cot \beta l$	Depends on length
SC, $0 < l < \lambda/4$	$+jX$	Inductor
SC, $\lambda/4 < l < \lambda/2$	$-jX$	Capacitor
OC, $0 < l < \lambda/4$	$-jX$	Capacitor
OC, $\lambda/4 < l < \lambda/2$	$+jX$	Inductor
SC, $l = \lambda/4$	∞	Open circuit
OC, $l = \lambda/4$	0	Short circuit
SC, $l = \lambda/2$	0	Short circuit
OC, $l = \lambda/2$	∞	Open circuit
Matched line	$Z_L = Z_0$	No reflection
Short circuit	$\Gamma = -1$	Complete reflection
Open circuit	$\Gamma = +1$	Complete reflection
Matched load	$\Gamma = 0$	$VSWR = 1$

MOST IMPORTANT FORMULAS TO MEMORIZE

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$\boxed{\gamma = \alpha + j\beta}$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right]}$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]}$$

$$\boxed{Z_{sc} = jZ_0 \tan \beta l}$$

$$\boxed{Z_{oc} = -jZ_0 \cot \beta l}$$

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\boxed{VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

$$\boxed{v_p = \frac{\omega}{\beta}}$$

$$\boxed{v_g = \frac{d\omega}{d\beta}}$$

$$\boxed{\lambda = \frac{v}{f}}$$

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} \quad \text{for lossless line}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}}$$

$$\boxed{\frac{R}{L} = \frac{G}{C} \quad \text{for distortionless line}}$$

$$\boxed{\alpha = \sqrt{RG}}$$

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

Made for Anand Sagar by Claude