UNIT-5 EMTL — SMITH CHART 10-MARK QUESTIONS WITH ANSWERS

Q1. Explain Smith Chart, its configuration and applications.

Answer:

A **Smith Chart** is a graphical tool used for the analysis of transmission lines. It represents all possible values of normalized impedance or admittance in terms of the **reflection coefficient**.

The reflection coefficient is represented as

$$\Gamma = \Gamma_r + j\Gamma_i$$

where,

$$\Gamma_r = \text{real part of reflection coefficient}$$

$$\Gamma_i = \text{imaginary part of reflection coefficient}$$

The normalized load impedance is

$$z_L = \frac{Z_L}{Z_0}$$

where,

$$Z_L = \text{load impedance}$$

$$Z_0 = \text{characteristic impedance of the line}$$

If the normalized impedance is written as

$$z = R + jX$$

then the Smith Chart gives the relation between impedance and reflection coefficient.

The basic relation between normalized impedance and reflection coefficient is

$$z = \frac{1+\Gamma}{1-\Gamma}$$

or,

$$R + jX = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

The Smith Chart consists of two families of circles:

1. **Constant resistance circles**
2. **Constant reactance circles**

The complete chart is formed by superimposing these two families of circles.

Configuration of Smith Chart

The Smith Chart is drawn in the Γ -plane. The horizontal axis represents Γ_r , and the vertical axis represents Γ_i .

The centre of the chart corresponds to

$$\Gamma = 0$$

This means that there is no reflection and the line is perfectly matched.

At the right extreme point of the chart,

$$\Gamma = 1$$

This corresponds to an open circuit.

At the left extreme point of the chart,

$$\Gamma = -1$$

This corresponds to a short circuit.

The outer circle of the Smith Chart represents

$$|\Gamma| = 1$$

This is the condition of total reflection.

Applications of Smith Chart

The Smith Chart is used for:

1. Calculation of Voltage Standing Wave Ratio, VSWR.
2. Finding positions of voltage maxima and voltage minima on a transmission line.
3. Calculation of impedance and admittance at any point on the transmission line.
4. Computing reflection coefficient for a given load impedance.
5. Impedance matching.
6. Single stub matching.
7. Double stub matching.
8. Transmission line impedance transformation.
9. Finding input impedance of short, open, quarter-wave and half-wave lines.

Thus, Smith Chart is a very useful graphical method for solving transmission line problems.

Q2. Derive the equation of constant resistance circles on Smith Chart.

Answer:

The normalized impedance is given by

$$z = R + jX$$

The reflection coefficient is

$$\Gamma = \Gamma_r + j\Gamma_i$$

The relation between normalized impedance and reflection coefficient is

$$z = \frac{1+\Gamma}{1-\Gamma}$$

Substituting,

$$R + jX = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i}$$

Multiplying numerator and denominator by the conjugate of denominator,

$$R + jX = \frac{(1 + \Gamma_r + j\Gamma_i)(1 - \Gamma_r + j\Gamma_i)}{(1 - \Gamma_r - j\Gamma_i)(1 - \Gamma_r + j\Gamma_i)}$$

The denominator becomes

$$(1 - \Gamma_r)^2 + \Gamma_i^2$$

The numerator becomes

$$1 - \Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i$$

Therefore,

$$R + jX = \frac{1 - \Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Comparing real parts,

$$R = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

Therefore,

$$R\left[(1-\Gamma_r)^2+\Gamma_i^2\right]=1-\Gamma_r^2-\Gamma_i^2$$

Expanding,

$$R\left[1+\Gamma_r^2-2\Gamma_r+\Gamma_i^2\right]=1-\Gamma_r^2-\Gamma_i^2$$

$$R+R\Gamma_r^2-2R\Gamma_r+R\Gamma_i^2=1-\Gamma_r^2-\Gamma_i^2$$

Bringing all terms to one side,

$$R\Gamma_r^2+\Gamma_r^2+R\Gamma_i^2+\Gamma_i^2-2R\Gamma_r+R-1=0$$

$$(R+1)\Gamma_r^2+(R+1)\Gamma_i^2-2R\Gamma_r=1-R$$

Dividing by $R+1$,

$$\Gamma_r^2+\Gamma_i^2-\frac{2R}{R+1}\Gamma_r=\frac{1-R}{R+1}$$

Completing the square,

$$\Gamma_r^2-\frac{2R}{R+1}\Gamma_r+\Gamma_i^2=\frac{1-R}{R+1}$$

$$\left(\Gamma_r-\frac{R}{R+1}\right)^2+\Gamma_i^2=\frac{1-R}{R+1}+\left(\frac{R}{R+1}\right)^2$$

Simplifying,

$$\left(\Gamma_r-\frac{R}{R+1}\right)^2+\Gamma_i^2=\frac{1}{(R+1)^2}$$

Hence, the equation of constant resistance circle is

$$\boxed{\left(\Gamma_r-\frac{R}{1+R}\right)^2+\Gamma_i^2=\left(\frac{1}{1+R}\right)^2}$$

Therefore,

$$\boxed{\text{Centre} = \left(\frac{R}{1+R}, 0\right)}$$

$$\boxed{\text{Radius} = \frac{1}{1+R}}$$

Important observations

1. The centres of all resistance circles lie on the Γ_r -axis.
2. The circle for $R = 0$ is the largest circle.
3. The circle for $R = \infty$ reduces to the point $(1, 0)$.
4. All resistance circles pass through the point $(1, 0)$, which represents open circuit.

Q3. Write the table of constant resistance circles and explain their nature.

Answer:

The equation of constant resistance circle is

$$\left(\Gamma_r - \frac{R}{1+R}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+R}\right)^2$$

Therefore,

$$\text{Centre} = \left(\frac{R}{1+R}, 0\right)$$

$$\text{Radius} = \frac{1}{1+R}$$

The values are:

Normalized Resistance R	Radius $\frac{1}{1+R}$	Centre $\left(\frac{R}{1+R}, 0\right)$
0	1	$(0, 0)$
$\frac{1}{2}$	$\frac{2}{3}$	$\left(\frac{1}{3}, 0\right)$
1	$\frac{1}{2}$	$\left(\frac{1}{2}, 0\right)$
2	$\frac{1}{3}$	$\left(\frac{2}{3}, 0\right)$
5	$\frac{1}{6}$	$\left(\frac{5}{6}, 0\right)$
∞	0	$(1, 0)$

Nature of constant resistance circles

The circle for $R = 0$ is centered at the origin and has radius 1.

$$R = 0 \Rightarrow \text{Centre} = (0, 0), \quad \text{Radius} = 1$$

This is the outer circle of the Smith Chart.

For $R = 1$,

$$\text{Centre} = \left(\frac{1}{2}, 0\right), \quad \text{Radius} = \frac{1}{2}$$

For $R = \infty$,

$$\text{Centre} = (1, 0), \quad \text{Radius} = 0$$

This is the open circuit point.

As R increases, the radius of the circle decreases. Hence, the constant resistance circles become progressively smaller and all of them are tangent at the point

$$(\Gamma_r, \Gamma_i) = (1, 0)$$

Thus, the resistance circles form one family of circles in the Smith Chart.

Q4. Derive the equation of constant reactance circles on Smith Chart.

Answer:

The normalized impedance is

$$z = R + jX$$

The reflection coefficient is

$$\Gamma = \Gamma_r + j\Gamma_i$$

The relation between normalized impedance and reflection coefficient is

$$z = \frac{1+\Gamma}{1-\Gamma}$$

Substituting,

$$R + jX = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

Multiplying numerator and denominator by the conjugate of denominator,

$$R + jX = \frac{(1+\Gamma_r + j\Gamma_i)(1-\Gamma_r + j\Gamma_i)}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Therefore,

$$R + jX = \frac{1-\Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Comparing imaginary parts,

$$X = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Therefore,

$$X\left[(1-\Gamma_r)^2+\Gamma_i^2\right]=2\Gamma_i$$

Dividing by X ,

$$(1-\Gamma_r)^2+\Gamma_i^2=\frac{2\Gamma_i}{X}$$

Expanding,

$$(1-\Gamma_r)^2+\Gamma_i^2-\frac{2\Gamma_i}{X}=0$$

Completing the square for Γ_i ,

$$(1-\Gamma_r)^2+\left(\Gamma_i^2-\frac{2\Gamma_i}{X}\right)=0$$

$$(\Gamma_r-1)^2+\left(\Gamma_i-\frac{1}{X}\right)^2=\frac{1}{X^2}$$

Hence, the equation of constant reactance circle is

$$\boxed{(\Gamma_r-1)^2+\left(\Gamma_i-\frac{1}{X}\right)^2=\left(\frac{1}{X}\right)^2}$$

Therefore,

$$\boxed{\text{Centre}=\left(1,\frac{1}{X}\right)}$$

$$\boxed{\text{Radius}=\frac{1}{|X|}}$$

Important observations

1. The centres of all reactance circles lie on the line

$$\Gamma_r=1$$

2. For $X > 0$, the circles lie above the Γ_r -axis.
3. For $X < 0$, the circles lie below the Γ_r -axis.
4. The $X = 0$ circle becomes the Γ_r -axis.
5. All reactance circles pass through the point $(1, 0)$, the open circuit point.
6. The reactance circles become smaller as $|X|$ increases.

Q5. Write the table of constant reactance circles and explain their nature.

Answer:

The equation of constant reactance circle is

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

Therefore,

$$\text{Centre} = \left(1, \frac{1}{X}\right)$$

$$\text{Radius} = \frac{1}{|X|}$$

The values are:

Normalized Reactance X	Radius $\frac{1}{\lvert X\rvert}$	Centre $(1, \frac{1}{X})$
0	∞	$(1, \infty)$
$\pm\frac{1}{2}$	2	$(1, \pm 2)$
± 1	1	$(1, \pm 1)$
± 2	$\frac{1}{2}$	$(1, \pm\frac{1}{2})$
± 5	$\frac{1}{5}$	$(1, \pm\frac{1}{5})$
$\pm\infty$	0	$(1, 0)$

Nature of constant reactance circles

For positive reactance,

$$X > 0$$

the reactance is inductive and the circles lie above the horizontal axis.

For negative reactance,

$$X < 0$$

the reactance is capacitive and the circles lie below the horizontal axis.

For zero reactance,

$$X = 0$$

the reactance circle becomes the horizontal axis itself.

As the magnitude of reactance increases,

$$|X| \to \infty$$

the radius tends to zero, and the circle ends at

$$(\Gamma_r, \Gamma_i) = (1, 0)$$

Therefore, all reactance circles pass through the open circuit point.

Q6. Explain the construction of Smith Chart using resistance and reactance circles.

Answer:

Smith Chart is constructed by plotting two families of circles in the reflection coefficient plane.

The reflection coefficient is

$$\Gamma = \Gamma_r + j\Gamma_i$$

The normalized impedance is

$$z = R + jX$$

The relation between normalized impedance and reflection coefficient is

$$z = \frac{1+\Gamma}{1-\Gamma}$$

From this relation, two important equations are obtained.

Constant resistance circle

$$\left(\Gamma_r - \frac{R}{1+R}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+R}\right)^2$$

Centre:

$$\left(\frac{R}{1+R}, 0\right)$$

Radius:

$$\frac{1}{1+R}$$

For different values of R , a family of circles is obtained.

Constant reactance circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2$$

Centre:

$$\left(1, \frac{1}{X}\right)$$

Radius:

$$\frac{1}{|X|}$$

For different values of X , another family of circles is obtained.

Construction procedure

1. Draw the Γ_r -axis and Γ_i -axis.
2. Draw the outer circle with centre at origin and radius 1.

$$|\Gamma| = 1$$

3. Plot constant resistance circles using

$$\text{Centre} = \left(\frac{R}{1+R}, 0\right)$$

and

$$\text{Radius} = \frac{1}{1+R}$$

4. Plot constant reactance circles using

$$\text{Centre} = \left(1, \frac{1}{X}\right)$$

and

$$\text{Radius} = \frac{1}{|X|}$$

5. The upper half of the chart represents inductive reactance.
6. The lower half of the chart represents capacitive reactance.
7. The centre of the chart represents matched load.

$$z = 1 + j0$$

8. The right extreme point represents open circuit.

$$z = \infty$$

9. The left extreme point represents short circuit.

$$z = 0$$

Thus, by combining constant resistance and constant reactance circles, the complete Smith Chart is obtained.

Q7. Explain impedance transformation on one-eighth, quarter-wave and half-wave transmission lines.

Answer:

For a lossless transmission line, the input impedance at a distance l from the load is given by

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

In normalized form,

$$z_{in} = \frac{z_L + j \tan \beta l}{1 + jz_L \tan \beta l}$$

where,

$$z_L = \frac{Z_L}{Z_0}$$

and

$$\beta = \frac{2\pi}{\lambda}$$

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1. One-eighth wave line

For one-eighth wave line,

$$l = \frac{\lambda}{8}$$

Therefore,

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8}$$

$$\beta l = \frac{\pi}{4}$$

$$\tan \beta l = \tan \frac{\pi}{4} = 1$$

Hence,

$$z_{in} = \frac{z_L + j}{1 + j z_L}$$

Therefore, for a one-eighth wave line,

$$\boxed{z_{in} = \frac{z_L + j}{1 + j z_L}}$$

On the Smith Chart, movement by $\lambda/8$ corresponds to rotation by 90° on the constant VSWR circle.

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2. Quarter-wave line

For quarter-wave line,

$$l = \frac{\lambda}{4}$$

Therefore,

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$\beta l = \frac{\pi}{2}$$

$$\tan \beta l = \tan \frac{\pi}{2} = \infty$$

Using

$$z_{in} = \frac{z_L + j \tan \beta l}{1 + j z_L \tan \beta l}$$

For $\tan \beta l = \infty$,

$$z_{in} = \frac{1}{z_L}$$

Thus,

$$\boxed{z_{in} = \frac{1}{z_L}}$$

In unnormalized form,

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

This is known as the impedance inverter property of a quarter-wave line.

On the Smith Chart, movement by $\lambda/4$ corresponds to rotation by 180° on the same VSWR circle.

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3. Half-wave line

For half-wave line,

$$l = \frac{\lambda}{2}$$

Therefore,

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$\beta l = \pi$$

$$\tan \beta l = \tan \pi = 0$$

Hence,

$$z_{in} = \frac{z_L + j0}{1 + jz_L(0)}$$

$$z_{in} = z_L$$

Therefore,

$$\boxed{z_{in} = z_L}$$

In unnormalized form,

$$\boxed{Z_{in} = Z_L}$$

Thus, a half-wave line repeats the load impedance at the input.

On the Smith Chart, movement by $\lambda/2$ corresponds to a complete rotation of 360° and returns to the same point.

Summary

Line Length	Electrical Length	Normalized Input Impedance
$\lambda/8$	45°	$z_{in} = \frac{z_L + j}{1 + jz_L}$
$\lambda/4$	90°	$z_{in} = \frac{1}{z_L}$
$\lambda/2$	180°	$z_{in} = z_L$

Q8. Explain impedance matching and its need in transmission lines.

Answer:

Impedance matching means making the load impedance equal to the characteristic impedance of the transmission line.

For perfect matching,

$$Z_L = Z_0$$

or in normalized form,

$$z_L = 1 + j0$$

When the load is matched to the line, the reflection coefficient becomes zero.

$$\Gamma = 0$$

The VSWR becomes

$$S = 1$$

and maximum power is delivered to the load.

Need for impedance matching

In a transmission line, impedance matching between source, line and load is one of the most important considerations.

If mismatch exists between the impedances, reflections occur on the transmission line.

These reflections cause standing waves, and as a result, maximum power is not delivered to the load.

Mismatch causes:

1. Reflected power.
2. Standing waves.
3. Increase in VSWR.
4. Loss of transmitted power.
5. Distortion and inefficient operation.

Therefore, impedance matching is used to eliminate reflections and deliver maximum power to the load.

Stub matching

A common method of impedance matching uses another transmission line called a **stub**.

A stub is a short length of transmission line connected either in series or shunt with the main transmission line.

The stub is usually terminated in either:

$$\text{Open circuit}$$

or

$$\text{Short circuit}$$

Stub matching may be:

1. Single stub matching.
2. Double stub matching.

The stub supplies the required reactance or susceptance to cancel the unwanted reactive part of the load or transformed load impedance.

Thus, impedance matching is achieved.

Q9. Explain different types of stub matching used in transmission lines.

Answer:

Stub matching is a technique used for impedance matching in transmission lines.

A stub is a short section of transmission line connected to the main transmission line. It is usually terminated either in an open circuit or in a short circuit.

The stub may be connected in two ways:

1. Series stub.
2. Shunt stub.

Also, depending on the termination, it may be:

1. Open-circuited stub.
2. Short-circuited stub.

Types shown in stub matching

The common stub arrangements are:

1. Series open stub.
2. Series short stub.
3. Shunt open stub.
4. Shunt short stub.

Series open stub

In this method, an open-circuited stub is connected in series with the main line.

The stub provides a series reactance.

It is used to cancel the unwanted reactance present in the transformed impedance.

Series short stub

In this method, a short-circuited stub is connected in series with the main transmission line.

It also provides a required series reactance.

Shunt open stub

In this method, an open-circuited stub is connected in shunt with the main line.

It provides a susceptance which cancels the unwanted susceptance at the point of connection.

Shunt short stub

In this method, a short-circuited stub is connected in shunt with the main line.

It is very commonly used in practical matching.

For shunt stub matching, it is better to use admittance rather than impedance.

The effective admittance at the point of connection is

$$y_{eff} = y_1 + y_2$$

For matching,

$$y_{eff} = 1$$

where,

$$y_1 = 1 + jb_1$$

and the stub must provide

$$y_2 = -jb_1$$

Therefore,

$$y_{eff} = 1 + jb_1 - jb_1$$

$$y_{eff} = 1$$

Hence, the transformed admittance becomes equal to the characteristic admittance of the line, and matching is obtained.

Q10. Explain single stub matching using Smith Chart.

Answer:

Single stub matching is a method of impedance matching in which a single transmission line section called a stub is connected either in series or shunt with the main line.

For shunt stub matching, the Smith Chart is used as an admittance chart.

The unknowns to be calculated are:

$$d_s = \text{distance of stub from load}$$

$$l_s = \text{length of stub}$$

Procedure

First, normalize the load impedance:

$$z_L = \frac{Z_L}{Z_0}$$

For shunt stub matching, convert it into normalized admittance:

$$y_L = \frac{1}{z_L}$$

Let

$$y_L = g + jb$$

Plot the point y_L on the Smith Chart.

Next, draw the constant VSWR circle passing through y_L .

Move from the load point towards the source along this VSWR circle until it intersects the unity conductance circle.

The unity conductance circle is

$$g = 1$$

Let the intersection point be

$$y_1 = 1 + jb_1$$

The distance moved from y_L to y_1 gives the distance of the stub from the load:

$$d_s$$

At the point of stub connection, the line admittance is

$$y_1 = 1 + jb_1$$

To obtain matching, the stub must provide an equal and opposite susceptance:

$$y_2 = -jb_1$$

Thus,

$$y_{eff} = y_1 + y_2$$

$$y_{eff} = 1 + jb_1 - jb_1$$

$$y_{eff} = 1$$

Therefore, the line is matched.

Calculation of stub length

For a short-circuited shunt stub, the point corresponding to the required susceptance

$$-jb_1$$

is marked on the periphery of the Smith Chart.

The distance measured from the short-circuit point to the susceptance point gives the stub length:

$$l_s$$

Therefore, in single stub matching:

$$\boxed{d_s = \text{distance from load to stub}}$$

$$\boxed{l_s = \text{length of stub}}$$

and the final matched condition is

$$\boxed{y_{eff} = 1}$$

Q11. Explain double stub matching.

Answer:

Double stub matching is another transmission line impedance matching technique. In this method, two stubs are connected to the main transmission line.

The stubs may be open-circuited or short-circuited, but short-circuited stubs are commonly preferred.

The main idea of double stub matching is to use two adjustable susceptances to obtain impedance matching.

Need for double stub matching

In single stub matching, both the position of the stub and the length of the stub are adjusted.

But in many practical cases, the stub position may be fixed.

In such situations, double stub matching is used.

Principle

For shunt double stub matching, the Smith Chart is used as an admittance chart.

The load admittance is first normalized:

$$y_L = \frac{Y_L}{Y_0}$$

The first stub adds a susceptance

$$jb_1$$

The admittance after first stub becomes

$$y_1' = y_L + jb_1$$

This admittance is then transformed through a fixed length of line between the two stubs.

At the second stub position, the admittance becomes

$$y_2'$$

The second stub adds another susceptance

$$jb_2$$

For matching, the final admittance must be

$$y_{eff} = 1 + j0$$

Therefore,

$$y_2' + jb_2 = 1 + j0$$

Hence,

$$jb_2 = -j\,\mathrm{Im}(y_2')$$

and the conductance at that point must become unity.

Features of double stub matching

1. It uses two stubs.
2. The distance between stubs is generally fixed.
3. The stub lengths are adjusted for matching.
4. It is useful when the stub location cannot be freely changed.
5. It is commonly solved using an admittance Smith Chart.
6. The final condition for matching is

$$\boxed{y_{eff} = 1 + j0}$$

Thus, double stub matching provides impedance matching by using two shunt or series stubs.

DERIVATIONS TO REMEMBER

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1. Reflection coefficient relation

$$z = \frac{1+\Gamma}{1-\Gamma}$$

$$\Gamma = \frac{z-1}{z+1}$$

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2. Resistance circle

$$\boxed{\left(\Gamma_r - \frac{R}{1+R}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+R}\right)^2}$$

$$\boxed{\text{Centre} = \left(\frac{R}{1+R}, 0\right)}$$

$$\boxed{\text{Radius} = \frac{1}{1+R}}$$

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3. Reactance circle

$$\boxed{(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2}$$

$$\boxed{\text{Centre} = \left(1, \frac{1}{X}\right)}$$

$$\boxed{\text{Radius} = \frac{1}{|X|}}$$

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4. Single stub matching condition

$$y_1 = 1 + jb_1$$

$$y_2 = -jb_1$$

$$y_{eff} = y_1 + y_2$$

$$y_{eff} = 1 + jb_1 - jb_1$$

$$\boxed{y_{eff} = 1}$$

PROBLEMS / SUMS AT THE END

Problem 1. Find the centre and radius of the constant resistance circle for $R = 2$.

Solution:

For constant resistance circle,

$$\text{Centre} = \left(\frac{R}{1+R}, 0\right)$$

$$\text{Radius} = \frac{1}{1+R}$$

Given,

$$R = 2$$

Therefore,

$$\text{Centre} = \left(\frac{2}{1+2}, 0\right)$$

$$\boxed{\text{Centre} = \left(\frac{2}{3}, 0\right)}$$

Radius:

$$\text{Radius} = \frac{1}{1+2}$$

$$\boxed{\text{Radius} = \frac{1}{3}}$$

Problem 2. Find the centre and radius of the constant resistance circle for $R = 5$.

Solution:

$$\text{Centre} = \left(\frac{R}{1+R}, 0\right)$$

$$\text{Radius} = \frac{1}{1+R}$$

Given,

$$R = 5$$

$$\text{Centre} = \left(\frac{5}{1+5}, 0\right)$$

$$\boxed{\text{Centre} = \left(\frac{5}{6}, 0\right)}$$

$$\text{Radius} = \frac{1}{1+5}$$

$$\boxed{\text{Radius} = \frac{1}{6}}$$

Problem 3. Find the centre and radius of the constant reactance circle for $X = 2$.

Solution:

For constant reactance circle,

$$\text{Centre} = \left(1, \frac{1}{X}\right)$$

$$\text{Radius} = \frac{1}{|X|}$$

Given,

$$X = 2$$

Therefore,

$$\text{Centre} = \left(1, \frac{1}{2}\right)$$

$$\boxed{\text{Centre} = \left(1, \frac{1}{2}\right)}$$

Radius:

$$\text{Radius} = \frac{1}{2}$$

$$\boxed{\text{Radius} = \frac{1}{2}}$$

Since $X = 2$ is positive, the circle lies above the Γ_r -axis.

Problem 4. Find the centre and radius of the constant reactance circle for $X = -1$.

Solution:

$$\text{Centre} = \left(1, \frac{1}{X}\right)$$

$$\text{Radius} = \frac{1}{|X|}$$

Given,

$$X = -1$$

Therefore,

$$\text{Centre} = \left(1, \frac{1}{-1}\right)$$

$$\boxed{\text{Centre} = (1, -1)}$$

Radius:

$$\text{Radius} = \frac{1}{|-1|}$$

$$\boxed{\text{Radius} = 1}$$

Since $X = -1$, the circle lies below the Γ_r -axis.

Problem 5. A transmission line has normalized load impedance $z_L = 2 + j1$. Find the input impedance for a quarter-wave line.

Solution:

For a quarter-wave line,

$$z_{in} = \frac{1}{z_L}$$

Given,

$$z_L = 2 + j1$$

Therefore,

$$z_{in} = \frac{1}{2 + j1}$$

Multiplying by conjugate,

$$z_{in} = \frac{2 - j1}{(2 + j1)(2 - j1)}$$

$$z_{in} = \frac{2 - j1}{2^2 + 1^2}$$

$$z_{in} = \frac{2 - j1}{5}$$

$$\boxed{z_{in} = 0.4 - j0.2}$$

Problem 6. A transmission line has normalized load impedance $z_L = 1 + j2$. Find the input impedance for a half-wave line.

Solution:

For a half-wave line,

$$z_{in} = z_L$$

Given,

$$z_L = 1 + j2$$

Therefore,

$$\boxed{z_{in} = 1 + j2}$$

Thus, a half-wave line repeats the load impedance at the input.

Problem 7. A transmission line has normalized load impedance $z_L = 2 + j1$. Find the input

impedance for a one-eighth wave line.

Solution:

For a one-eighth wave line,

$$z_{in} = \frac{z_L + j}{1 + jz_L}$$

Given,

$$z_L = 2 + j1$$

Therefore,

$$z_{in} = \frac{2 + j1 + j}{1 + j(2 + j1)}$$

$$z_{in} = \frac{2 + j2}{1 + j2 + j^2}$$

Since,

$$j^2 = -1$$

$$z_{in} = \frac{2 + j2}{1 + j2 - 1}$$

$$z_{in} = \frac{2 + j2}{j2}$$

$$z_{in} = \frac{2}{j2} + \frac{j2}{j2}$$

$$z_{in} = \frac{1}{j} + 1$$

Since,

$$\frac{1}{j} = -j$$

$$\boxed{z_{in} = 1 - j}$$

Problem 8. A normalized admittance at the point of stub connection is $y_1 = 1 + j1.5$. Find the required stub admittance for matching.

Solution:

For shunt stub matching,

$$y_{eff} = y_1 + y_2$$

For matching,

$$y_{eff} = 1 + j0$$

Given,

$$y_1 = 1 + j1.5$$

Therefore, the stub must provide opposite susceptance:

$$y_2 = -j1.5$$

Now,

$$y_{eff} = 1 + j1.5 - j1.5$$

$$y_{eff} = 1 + j0$$

$$\boxed{y_2 = -j1.5}$$

Thus, the required stub admittance is

$$\boxed{-j1.5}$$

Problem 9. A normalized admittance at the stub point is $y_1 = 1 - j0.8$. Find the required stub admittance.

Solution:

For matching,

$$y_{eff} = 1 + j0$$

Given,

$$y_1 = 1 - j0.8$$

Let the required stub admittance be

$$y_2 = jb$$

Then,

$$y_{eff} = y_1 + y_2$$

$$1 + j0 = 1 - j0.8 + jb$$

Therefore,

$$-j0.8 + jb = 0$$

$$jb = j0.8$$

Hence,

$$\boxed{y_2 = j0.8}$$

Problem 10. If $Z_0 = 50\Omega$ and $Z_L = 100 + j50\Omega$, find normalized load impedance.

Solution:

Normalized impedance is

$$z_L = \frac{Z_L}{Z_0}$$

Given,

$$Z_0 = 50\Omega$$

$$Z_L = 100 + j50\Omega$$

Therefore,

$$z_L = \frac{100 + j50}{50}$$

$$z_L = \frac{100}{50} + j\frac{50}{50}$$

$$\boxed{z_L = 2 + j1}$$

Problem 11. If normalized load impedance is $z_L = 2 + j1$, find normalized load admittance.

Solution:

Normalized admittance is

$$y_L = \frac{1}{z_L}$$

Given,

$$z_L = 2 + j1$$

Therefore,

$$y_L = \frac{1}{2 + j1}$$

Multiplying numerator and denominator by conjugate,

$$y_L = \frac{2 - j1}{(2 + j1)(2 - j1)}$$

$$y_L = \frac{2 - j1}{2^2 + 1^2}$$

$$y_L = \frac{2 - j1}{5}$$

$$\boxed{y_L = 0.4 - j0.2}$$

MOST IMPORTANT FINAL EXAM POINTS

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1. Learn the derivation of constant resistance circles.
2. Learn the derivation of constant reactance circles.
3. Remember the centre and radius formulas.
4. Remember that $X > 0$ lies above the axis and $X < 0$ lies below the axis.
5. Remember that the centre of the Smith Chart is matched load.
6. Remember that the right extreme point is open circuit.
7. Remember that the left extreme point is short circuit.
8. For shunt stub matching, use admittance chart.
9. For single stub matching:

$$y_1 = 1 + jb_1$$

$$y_2 = -jb_1$$

$$y_{eff} = 1$$

10. For impedance transformation:

$$\lambda/8: \quad z_{in} = \frac{z_L + j}{1 + jz_L}$$

$$\lambda/4: \quad z_{in} = \frac{1}{z_L}$$

$$\lambda/2: \quad z_{in} = z_L$$

Made for Anand Sagar by Claude